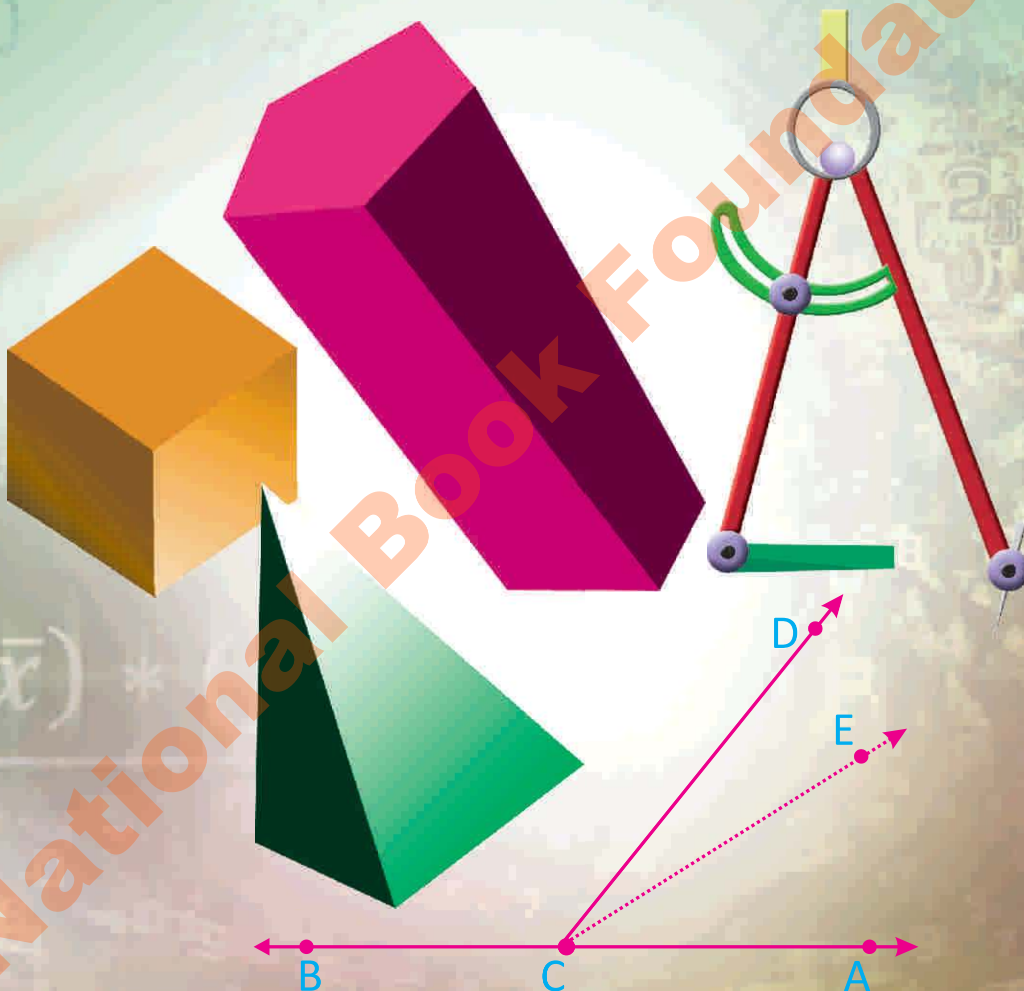
Textbook of

MATHEMATICS

GRADE 8



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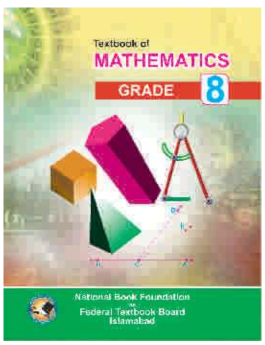

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Preface

MATHEMATICS GRADE- 8 is developed according to the National Curriculum 2006 and National Style Guide. It is being published since 2014 and in 2015 it was presented under the new management and supervision of textbook development principles and guidelines with new design and layout.

Mathematics helps students to take responsibility for what they think while this objective applies to all subjects. It is an area in which even young children can solve problems and have confidence over it. Mathematics has been playing an important role in school education since the beginning of formal education. It is not only played a dominant role in the advancement of civilization but also in the development of physical sciences and other disciplines. The importance of teaching Mathematics at middle school level are as under:

1. To develop understanding of modern mathematical concepts.
2. To enable the student to solve mathematical problems which have a practical value in real life situations.
3. To enable the student to develop the ability to measure and construct geometric figures.

Like before, the National Book Foundation has made specific endeavours to publish the text and illustrations in much effective form. The meticulous effort of the team is acknowledged.

Our efforts are to make textbooks teachable with quality, i.e. maintaining of standards. It is a continuous effort and we will get feedback of the yearly feasibility reports and redesign the textbook every year. The text items given in the exercises are for learning reinforcement. The examination questions are to be prepared according to the SLO's and the Bloom's Taxonomy.

Quality of Standards, Pedagogical Outcomes, Taxonomy Access and Actualization of Style is our motto. With these elaborations, this series of new development was presented for use. After educational feedback, research and necessary changes, the book is being published again.

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بِسۡمِ اللّٰہِ الرَّحۡمٰنِ الرَّحِیۡمِ

شروع اللہ کے نام سے جو بڑا مہربان، نہایت رحم والا ہے

Contents

1

OPERATION ON SETS

This is 14 days unit (periods including homework)

After studying this unit, you will be able to:

- recognize the following sets:
  - N = Set of natural numbers,
  - W = Set of whole numbers
  - Z = Set of integers,
  - E = Set of even numbers
  - O = Set of odd numbers,
  - P = Set of prime numbers
  - Q = Set of rational numbers
- find a subset of a set.
- define proper and improper subsets of a set.
- find power set of a set.
- verify commutative and associative laws with respect to union and intersection.
- verify the distributive laws.
- state and verify De Morgan's laws.
- demonstrate union and intersection of three overlapping sets through Venn diagrams.
- verify associative and distributive laws through Venn diagrams.

Reading

Set:

It is collection of well defined and distinct objects e.g.

A = {1, 2, 3, 4, 5} , B = {a, e, i}

Empty or Null or Void Set:

A set having no element is called empty set. It is denoted by ϕ.

Thus ϕ = { }

Singleton Set:

A set having ognly one element is called a singleton set. e.g.

E = {3}, F = {m}

Finite and Infinite Set:

The set whose elements are limited is called a finite set. The set which is not finite is called an infinite set. e.g.

{1, 3, 5, 7, 9} is finite set and {1, 2, 3, 4,...} is an infinite set.

Equal and Equivalents Sets:

Two sets A and B are equal if every element of A is present in B and every element of B is present in A. e.g.

A = {5, 7, 9}, B = {9, 5, 7} are equal sets. We write A = B

Two sets P and Q are equivalent if both have same number of elements. e.g. The sets P = {a, e, i, o, u} and Q = {2, 3, 4, 5, 6,} are equivalent sets.

Note: Equal sets are also equivalent.

1.1 SETS

1.1.1 Most Commonly used Sets of Numbers

Plus the Memory
- The symbol $\vee$ is used for "or"
- The symbol $\wedge$ is used for "and".

(1) N = Set of natural numbers = {1,2 ,3, 4,…}
(2) W = Set of whole numbers = {0, 1, 2, 3, 4,…}
(3) Z = Set of integers = {0, $\pm$1, $\pm$2, $\pm$3,...}
(4) E = Set of even numbers = {0, $\pm$2, $\pm$4, $\pm$6,…}
(5) O = Set of odd numbers = {$\pm$1, $\pm$3, $\pm$5,…}
(6) P = Set of prime numbers = {2, 3, 5, 7, 11, 13,…}
(7) Q = Set of rational numbers = $\left\{\frac{p}{q} : p, q \in Z \wedge q \neq 0\right\}$

The above mentioned sets from (1)-(6) are written in both descriptive method and tabular form (Roster method). However the set Q is first written in descriptive method then in set builder form.

1.1.2 Subset of a Set

If A and B are two sets then the set A is called a subset of a set B if each element of the set A is contained in the set B. Symbolically we write $A \subseteq B$ and read it as " A is a subset of B".

For example if A = {1, 2, 3, 4} and B= {1, 2, 3, 4, 5, 6} then $A \subseteq B$ because all elements of the set A are also the elements of the set B.

Note: (i) Every set is subset of itself. e.g. If A is any set then $A \subseteq A$.

(ii) Empty set is subset of every set. e.g. For any set A, $\phi \subseteq A$.

(iii) If a set A has n elements, then the numbers of subsets of A are 2^n.

Example 1:

Find any four subsets of the set {3, 5, 7}.

Solution:

Four subsets of {3, 5, 7} are:

{3}, {5}, {7}, {3, 5}

Example 2:

Find all subsets of the set B = {a, c, e, g}.

Plus the Memory
If a set A has 5 elements, then we write n(A) = 5

Solution:

All subsets of the set B are:

{ }, {a}, {c}, {e}, {g}, {a, c}, {a, e}, {a, g}, {c, e}, {c, g}, {e, g}, {a, c, e}, {a, c, g}, {a, e, g}, {c, e, g}, {a, c, e, g}

Note: (i) The set of natural numbers is the subset of whole numbers.

(ii) $N \subseteq W \subseteq Z \subseteq Q$

1.1.3 Types of Subset

There are two types of subsets:

(i) Proper subset (ii) Improper subset

(i) Proper Subset

If each element of the set A is contained in the set B and there is at least one element of B which is not an element of the set A then the set A is called proper subset of the set B. Symbolically we write $A \subset B$ and read it as " A is proper subset of B".

For example the set N is proper subset of the set W because 0 is an element of W which is not contained in N.

(ii) Improper Subset

If each element of the set A is contained in the set B and there is no element of the set B which is not an element of the set A then the set A is called an improper subset of the set B. Symbolically we write $A \subseteq B$ and read it as " A is improper subset of B".

e.g. {0, 1, 2, 3} is improper subset of {0, 3, 1, 2}.

Example 3:

Write five proper subsets and one improper subset of the set C = {2, 3, 5, 7}. How many proper subsets of C are in total?

Solution:

Five proper subsets of C are:

{2}, {3}, {2, 3}, {2, 5}, {3, 5, 7} etc.

Improper subset of C is {2, 3, 5, 7}.

Number of proper subsets of C are 16 – 1 = 15.

Note: (i) ϕ is proper subset of every non empty set.

(ii) Every set is an improper subset of itself.

1.1.4 Power Set of a Set

The power set of any set is the set consisting of all possible subsets of it. If A is the given set then power set of A is denoted by P(A).

For example if A = {0, 2}, then possible subsets of the set A are ϕ, {0}, {2}, {0, 2}.

$\therefore$ P (A) = {ϕ, {0}, {2}, {0, 2}}

Formula for finding the numbers of subsets in a power set is 2^n where n is number of elements in the given set.

For the above set P, n = 4.

Therefore number of subset in power set = $2^4 = 16$

Example 4:

How many subsets does the set B has if B = {0, 1, 2}. Also find the power set of B.

Solution:

B = {0, 1, 2}.

Here n = 3.

Number of subsets of the set B are $2^3 = 8$.

All subsets of B are:

{ }, {0}, {1}, {2}, {0, 1}, {0, 2}, {1, 2}, {0, 1, 2}

$\therefore$ P(B) = {{ }, {0}, {1}, {2}, {0, 1}, {0, 2}, {1, 2}, {0, 1, 2}}

Example 5:

If X = {e, f, g, h}, then find P(X).

Solution:

All subsets of the set X are:

ϕ, {e}, {f}, {g}, {h}, {e, f}, {e, g}, {e, h}, {f, g}, {f, h}, {g, h}, {e, f, g}, {e, f, h}, {e, g, h}, {f, g, h}, {e, f, g, h}

P(X) = {ϕ, {e}, {f}, {g}, {h}, {e, f}, {e, g}, {e, h}, {f, g}, {f, h}, {g, h}, {e, f, g}, {e, f, h}, {e, g, h}, {f, g, h}, {e, f, g, h}}

Note: (i) Elements of power sets are the sets.

(ii) Power set of an empty set is never empty.

Exercise 1.1

1. Find three subsets of the following sets.
   (i) {2, 4} (ii) {a, c, e }
2. Find all possible subsets of the following sets.
   (ii) {–1, 0, 1} (ii) { } (iii) { m, n, o, p}
3. Write four proper subsets and one improper subset of the following sets.
   (i) {–1, –2, –3} (ii) $\left\{\frac{1}{2}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}\right\}$ (iii) {pen, pencil, copy}
4. Name the set which has
   (i) Only one subset.
   (ii) Only one proper subset.
   (iii) No proper subset.
5. Write the number of elements in the power set of following sets.
   (i) {1, 3, 5, 7, 9} (ii) {0, 1, 2, 3, 4, 5 } (iii) ϕ
6. Find the power set of the following sets.
   (i) {a, b} (ii) {0, 2, 4} (iii) {1, 2, 3, 4}
   (iv) {101} (v) ϕ

Reading

1.2 OPERATIONS ON SETS

1.2.1 Commutative Law of Union

Let A and B are any two sets, then $A \cup B = B \cup A$ is always true. This is called commutative property of union of two sets.

Example 6:

If A = {2, 3, 5, 7, 11}, B = {1, 4, 9, 16}, then verify the commutative property of union for A and B.

Solution:

Here, A = {2, 3, 5, 7, 11}, B = {1, 4, 9, 16}.

Then $A \cup B = \{1, 2, 3, 4, 5, 7, 9, 11, 16\}$

$B \cup A = \{1, 2, 3, 4, 5, 7, 9, 11, 16\}$.

$\therefore$ $A \cup B = B \cup A$ or commutative property of union of two sets is verified.

1.2.2 Commutative Law of Intersection

Let A and B be any two sets, then $A \cap B = B \cap A$ is always true. This is called commutative property of intersection of sets.

Example 7:

If A = Set of first five whole multiples of 4 and B = {x : x is factor of 16}, then verify commutative property of intersection for A and B.

Solution:

Here A = {0, 4, 8, 12, 16} and B = {1, 2, 4, 8,16 }.

Then $A \cap B$ = {4, 8, 16} and $B \cap A$ = {4, 8, 16}.

As $A \cap B = B \cap A$, so commutative property of intersection is verified.

1.2.3 Associative Law of Union

Let A, B and C be any three sets, then $(A \cup B) \cup C = A \cup (B \cup C)$ is always true. This property is called associative property of union of sets.

Example 8:

If A = {–2, –1, 0, 1}, B = {–1, 0, 1, 2} and C = C = {–2, –1, 0}, then verify the associative property of union for A, B and C.

Solution:

Here A = {–2, –1, 0, 1}, B = {–1, 0, 1, 2} and C = {–2, –1, 0}.

Then $(A \cup B) \cup C$ = ({–2, –1, 0, 1} ∪ {–1, 0, 1, 2}) ∪ {–2, –1, 0}

= {–2, –1, 0, 1, 2} ∪ {–2, –1, 0}

= {–2, –1, 0, 1, 2}

Also $A \cup (B \cup C)$ = {–2, –1, 0, 1} ∪ ({–1, 0, 1, 2} ∪ {–2, –1, 0})

= {–2, –1, 0, 1} ∪ {–2, –1, 0, 1, 2}

= {–2, –1, 0, 1, 2}

As $(A \cup B) \cup C = A \cup (B \cup C)$, so associative property of union is verified.

1.2.3 Associative Law of Intersection

Let A, B and C be any three sets then $(A \cap B) \cap C = A \cap (B \cap C)$, is always true. This is called associative property of intersection of sets.

Example 9:

If A = {a, c, e, h, i, m, s, t}, B = {e, g, m, o, r, t, y} and C = {a, b, e, g, l, r}, Then verify associative property of intersection for A, B and C.

Solution:

Here, A = {a, c, e, h, i, m, s, t},
B = {e, g, m, o, r, t, y} and
C = {a, b, e, g, l, r}, then

$(A \cap B) \cap C = (\{a, c, e, h, i, m, s, t\} \cap \{e, g, m, o, r, t, y\}) \cap \{a, b, e, g, l, r\}$
$= \{e, m, t\} \cap \{a, b, e, g, l, r\}$
$= \{e\}$

$A \cap (B \cap C) = (a, c, h. e, i, m, s, t\} \cap (\{e, g, m, o ,r, t, y\} \cap \{a, b, e, g, l, r\})$
$= \{a, c, e, h, i, m, s, t\} \cap \{e, g, r\}$
$= \{e\}$

As $(A \cap B) \cap C = A \cap (B \cap C)$, so the associative property of intersection is verified.

Exercise 1.2

1. Verify commutative property of union for the following pair of sets.
   A = {2, 3, 5, 7, 11}, B = {5, 6, 7, 8, 9, 10}
2. Verify commutative property of intersection for the following pair of sets.
   X = {s, c, i, e, n}, Y = {m, a, t, h, e, i, c, s}
3. If M = Set of vowels in English alphabets,
   N = Set of consonants in English alphabets.
   Verify that: (i) $M \cup N = N \cup M$ (ii) $M \cap N = N \cap M$
4. Verify associative property of union for the following sets.
   (i) A = {0, 1, 2, 3, …,10}, B = {1, 2, 3, 4, 5} and C = {0, 2, 4, 6, 8}

(ii) P = $\left\{\frac{1}{2},\frac{2}{3},\frac{3}{4},\frac{4}{5}\right\}$, Q = $\left\{-\frac{1}{2},-\frac{2}{3},-\frac{3}{4},-\frac{4}{5}\right\}$ and R = $\left\{\frac{2}{3},\frac{3}{4},-\frac{2}{3},-\frac{3}{4}\right\}$

5. Verify associative property of intersection for the following sets.
   (i) A = {0, 1, 2, 3, 4, 5}, B = {–5, –4, –3, –2, –1}
   and C = {–2, –1, 0, 1, 2 , 3}
   (ii) X = {2, 3, 5, 7, 11, 13, 17, 19}, Y = {1, 3, 5, …, 19}
   and Z = {2, 4, 6, …,20}.
6. If X = {1, 2, 3,…,10}, Y = {0, 2, 4, 6, 8, 10}
   and Z = {0, 1, 2, 3,…10}, then prove that:
   (i) $(X\cup Y) \cup Z = X\cup(Y \cup Z)$
   (ii) $(X\cap Y)\cap Z = X\cap(Y\cap Z)$

Reading

1.2.5 Distributive Laws

(a) Distributive Law of Union over Intersection

If A, B and C be any three sets then distributive law of union over intersection is stated as

$$A\cup(B\cap C) = (A\cup B)\cap(A\cup C)$$

Example 10: If A = {1, 3, 5, 7}, B = {2, 3, 5, 7}, C = {1, 2, 3, 4, 5} then verify that $A\cup(B\cap C) = (A\cup B)\cap(A\cup C)$.

Solution:

Here A = {1, 3, 5, 7}, B = {2, 3, 5, 7}, C = {1, 2, 3, 4, 5}

L.H.S = $A\cup(B\cap C)$

= {1, 3, 5, 7} ∪ ({2, 3, 5, 7} ∩ {1, 2, 3, 4, 5})

= {1, 3, 5, 7} ∪ {2, 3, 5} = {1, 2, 3, 5, 7}

R.H.S = $(A\cup B)\cap(A\cup C)$

= ({1, 3, 5, 7} ∪ {2, 3, 5, 7}) ∩ ({1, 3, 5, 7} ∪ {1, 2, 3, 4, 5})

= {1, 2, 3, 5, 7} ∩ {1, 2, 3, 4, 5, 7} = {1, 2, 3, 5, 7}

Hence $A\cup(B\cap C) = (A\cup B)\cap(A\cup C)$.

(b) Distributive Law of Intersection over Union

If A, B and C be any three sets then distributive law of intersection over union is stated as

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

Example 11: If $A = \{a, b, c, d, e\}$, $B = \{a, e, i, o\}$, $C = \{b, d, e, f\}$ then verify that $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

Solution:

Here $A = \{a, b, c, d, e\}$, $B = \{a, e, i, o\}$, $C = \{b, d, e, f\}$

L.H.S $= A \cap (B \cup C)$

$= \{a, b, c, d, e\} \cap (\{a, e, i, o\} \cup \{b, d, e, f\})$

$= \{a, b, c, d, e\} \cap \{a, b, d, e, f, i, o\}$

$= \{a, b, d, e\}$

L.H.S $= (A \cap B) \cup (A \cap C)$

$= (\{a, b, c, d, e\} \cap \{a, e, i, o\}) \cup (\{a, b, c, d, e\} \cap \{b, d, e, f\})$

$= \{a, e\} \cup \{b, d, e\} = \{a, b, d, e\}$

Hence $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

1.2.6 De Morgan's Laws

De Morgan's laws are named after the British mathematician and logician Augustus De Morgan(1806-1871). These laws relate the three basic set operations to each other; the union, the intersection and the complement.

De Morgan's laws are stated as

(i) $(A \cup B)' = A' \cap B'$ **(ii)** $(A \cap B)' = A' \cup B'$

Example 12:

If $U = \{a, b, c, d, \ldots, h\}$, $A = \{a, b, c, d\}$, $B = \{e, f, g, h\}$, then prove that

(i) $(A \cup B)' = A' \cap B'$ **(ii)** $(A \cap B)' = A' \cup B'$

Solution:

(i) $(A \cup B)' = A' \cap B'$

Here $U = \{a, b, c, d, e, f, g, h\}$, $A = \{a, b, c, d\}$, $B = \{e, f, g, h\}$

L.H.S $= (A \cup B)'$

$= U - (A \cup B)$

$= \{a, b, c, d, \ldots, h\} - (\{a, b, c, d\} \cup \{e, f, g, h\})$

$= \{a, b, c, d, \ldots, h\} - \{a, b, c, d, \ldots, h\} \quad = \phi$

R.H.S = $A' \cap B'$

$= (U - A) \cap (U - B)$

$= (\{a, b, c, d, \ldots, h\} - [a, b, c, d\}) \cap (\{a, b, c, d, \ldots, h\} - [e, f, g, h\})$

$= \{e, f, g, h\} \cap \{a, b, c, d\} \quad = \phi$

L.H.S = R.H.S

$\therefore \quad (A \cup B)' = A' \cap B'$

(ii) $(A \cap B)' = A' \cup B'$

L.H.S = $(A \cap B)'$

$= U - (A \cap B)$

$= \{a, b, c, d, \ldots, h\} - (\{a, b, c, d\} \cap \{e, f, g, h\}$

$= \{a, b, c, d, \ldots, h\} - \{\ \}$

$= \{a, b, c, d, \ldots, h\}$

R.H.S = $A' \cup B'$

$= (U - A) \cup (U - B)$

$= (\{a, b, c, \ldots, h\} - \{e, f, g, h\}) \cup (\{a, b, c, \ldots, h\} - \{e, f, g, h\})$

$= \{e, f, g, h\} \cup \{a, b, c, d\}$

$= \{a, b, c, d, \ldots, h\}$

L.H.S = R.H.S

$\therefore \quad (A \cup B)' = A' \cap B'$

Exercise 1.3

1. Prove that

(a) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(b) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

when

(i) $A = \{0,1,2,3\}$, $B = \{2,3,4,5,6\}$, $C = \{5,6,7,8,9,10\}$

(ii) $A = \{l, m, n, o, p, q\}$, $B = \{r, s, t, u\}$, $C = \{t, u, v, w\}$

(iii) $A = \{+, -, \times\}$, $B = \{-, \times, \div\}$, $C = \{-, \div, \sqrt{\ }\}$

2. Verify distributive law of union over intersection for the following sets.

$P = \{1, 2, 3, ...\}$, $Q = \{0, 1, 2, 3, ...\}$, $R = \{0, \pm 1, \pm 2, \pm 3, ...\}$

3. Verify distributive law of intersection over union for the following sets.

$X = \{\ \}$, $Y = \{0\}$, Z = Set of natural numbers

4. If $U = \{1, 2, 3, \ldots, 10\}$, $A = \{2, 3, 5, 7, 9\}$, $B = \{2, 4, 6, 8, 10\}$

Then prove that

(i) $(A \cup B)' = A' \cap B'$ **(ii)** $(A \cap B)' = A' \cup B'$

5. Verify De Morgan's laws for the following sets.

$U = \{x : x \in W \wedge 0 \leq x \leq 20\}$, $C = \{x : x \in E \wedge 0 \leq x \leq 20\}$

and $D = \{x : x \in O \wedge 1 \leq x \leq 19\}$.

Reading

1.3 VENN DIAGRAMS

1.3.1 Overlapping Sets

Recall that two sets A and B are called overlapping if

(i) There is at least one element common in both the sets.

(ii) The set A and the set B are not subsets of each other.

For example the sets

$A = \{1, 3, 5, 7, 9\}$ and $B = \{1, 2, 3, 4, 5\}$ are overlapping as

$A \cap B = \{1, 3, 5\} \neq \phi$, and A and B are not subsets of each other.

1.3.2 Union and Intersection of Three Overlapping Sets through Venn Diagram

We have learnt about union and intersection of two overlapping sets through Venn diagram in grade VII. Let us revise it by taking two overlapping sets as follows.

Consider $A = \{1, 2, 3, 4, 5\}$, $B = \{4, 5, 6, 7, 8\}$

Venn diagram of A and B is shown in the figure 1.

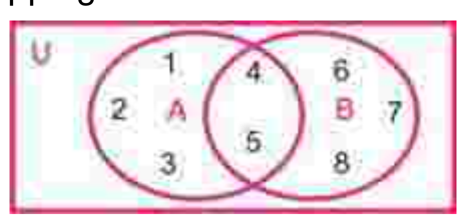


Figure 1

As $A \cap B = \{4, 5\}$ i.e. $A \cap B$ represents the set containing the common elements. Therefore Venn diagram showing $A \cap B$ is given in figure 2.

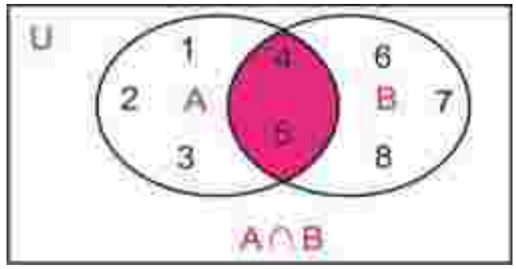


Figure 2

Again $A \cup B = \{1, 2, 3,..., 8\}$ i.e. $A \cup B$ represents the set containing all elements of A and B. Therefore Venn diagram showing $A \cup B$ is given in figure 3.

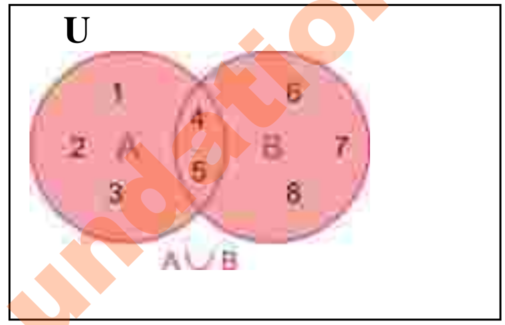


Figure 3

We solve some examples to understand the concept of union and intersection of three overlapping sets through Venn diagram.

Example 13:

Represent the following sets using Venn diagram.
$A = \{0, 1, 2, 3, 4\}, B = \{3, 4, 5, 6\}, C = \{6, 7, 8, 9\}$

Solution:

Representation using Venn diagram is shown in the following figure.

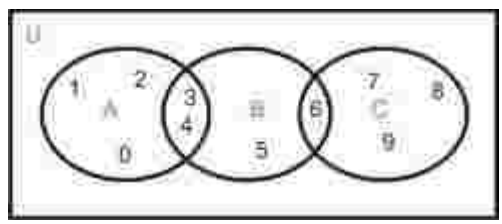


Figure 4

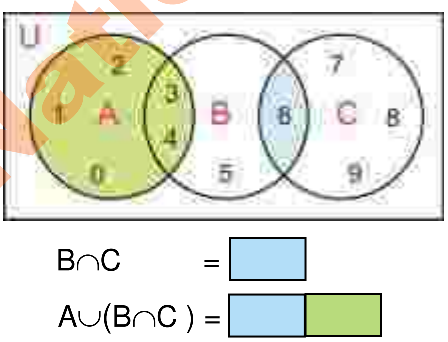
Example 14:

Given sets A, B, C in example 13.

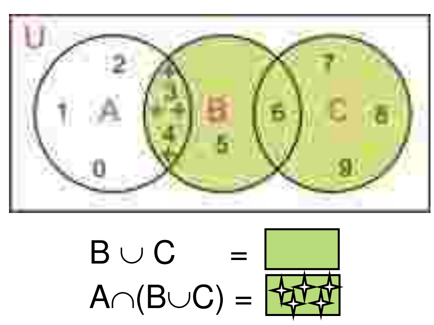
Find $A \cup (B \cap C)$, $A \cap (B \cup C)$ using Venn diagram.

Solution:

(i)



(ii)



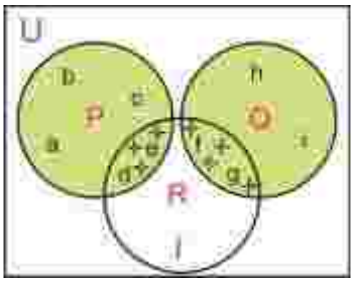
Example 15:

If P = {a, b, c, d, e}, Q = {f, g, h, i}, R = {d, e, f, g, j}.

Find (P ∪ Q) ∩ R and (P ∩ Q) ∪ R through Venn diagram.

Solution:

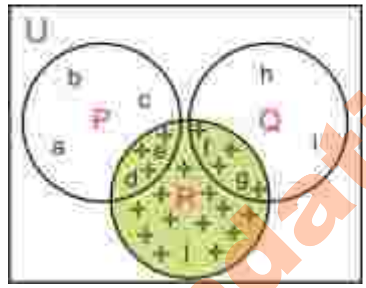
(i)



P ∪ Q =

(P∪Q) ∩R =

(ii)



P ∩ Q =

(P ∩Q) ∪R =

1.3.3 Verification of Associative Law through Venn Diagram

We illustrate the concept with the help of following examples.

(a) Associative Property of Union.

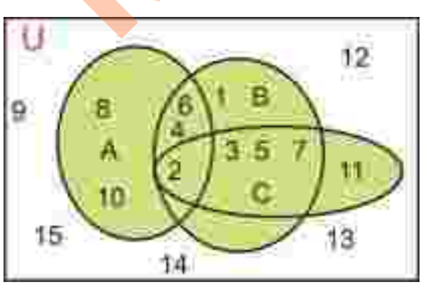
(A∪ B) ∪ C = A∪ (B∪ C)

Example 16:

If U = {1, 2, 3, 4, 5,..., 15}, A = {2, 4, 6, 8, 10}, B = {1, 2, 3, 4, 5, 6, 7} and C = {2, 3, 5, 7, 11}, then verify the associative property of union with the help of Venn diagram.

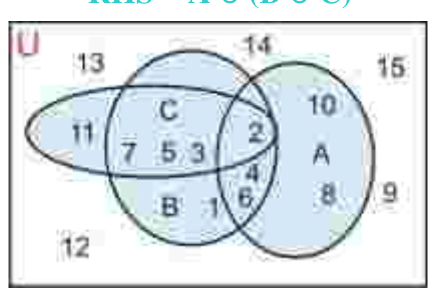
Solution:

LHS = (A ∪ B) ∪ C



(A∪B) ∪ C =

RHS = A ∪ (B ∪ C)



A∪ (B ∪ C) =

From both the figures, it is observed that same region is shaded.

i.e. $(A \cup B) \cup C = A \cup (B \cup C)$.

$\therefore$ Associative property of Union is verified.

(b) Associative Property of Intersection

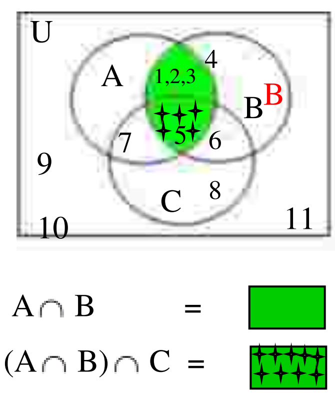
$$(A \cap B) \cap C = A \cap (B \cap C)$$

Example 17:

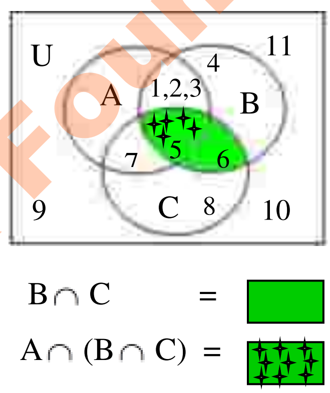
If U = {1, 2, 3,…,11}, A = {1, 2, 3, 5, 7}, B = {1, 2, 3, 4, 5, 6} and C = {5, 6, 7, 8} then verify the associative property of intersection.

Solution:

LHS = $(A \cap B) \cap C$



RHS = $A \cap (B \cap C)$



From both the figures it is observed that same regions are shaded.

i.e. $(A \cap B) \cap C = A \cap (B \cap C)$

$\therefore$ Associative property of intersection is verified.

1.3.4 Verification of Distributive Law through Venn Diagram

(a) Distributive Property of Union over Intersection

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

(b) Distributive Property of Intersection over Union

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

Example 18: Verify using Venn diagram

(a) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(b) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

When $U = \{x : x \in W \wedge x \leq 12\}$,

$A = \{x : x \in W \wedge x \leq 4\}$

$B = \{y : y \in N \wedge 3 \leq y \leq 7\}$ and

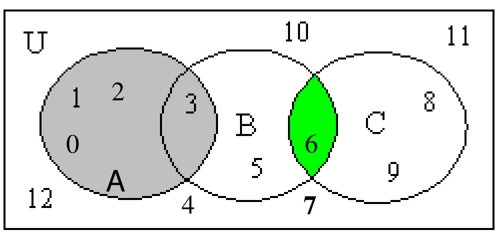
$C = \{z : z \in N \wedge 3 \leq z \leq 7\}$.

> ***Think Hard***
> In a class of 50 students, 18 read biology, 26 take mathematics, and 2 take both biology and mathematics. How many students in the class are not enrolled in either biology or mathematics? Show the result with the help of diagram.

Solution:

(a) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

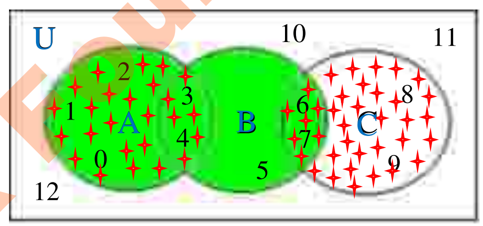
LHS = A ∪ (B ∩ C)



B ∩ C =

A ∪ (B ∩ C) =

RHS = (A∪B) ∩ (A∪C)



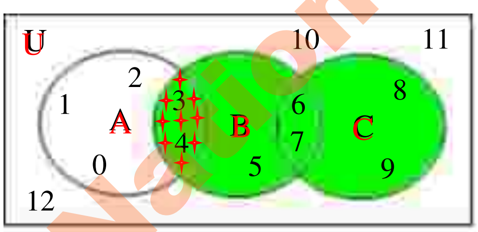
A ∪ B =

A ∪ C =

(A ∪ B) ∩ (A ∪ C) =

(b) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C$

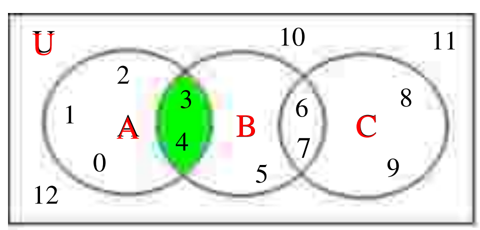
LHS = A ∩ (B ∪ C)



B ∪ C =

A ∩ (B ∪ C) =

RHS = (A ∩ B) ∪ (A ∩ C)



A ∩ B = , A ∩ C =

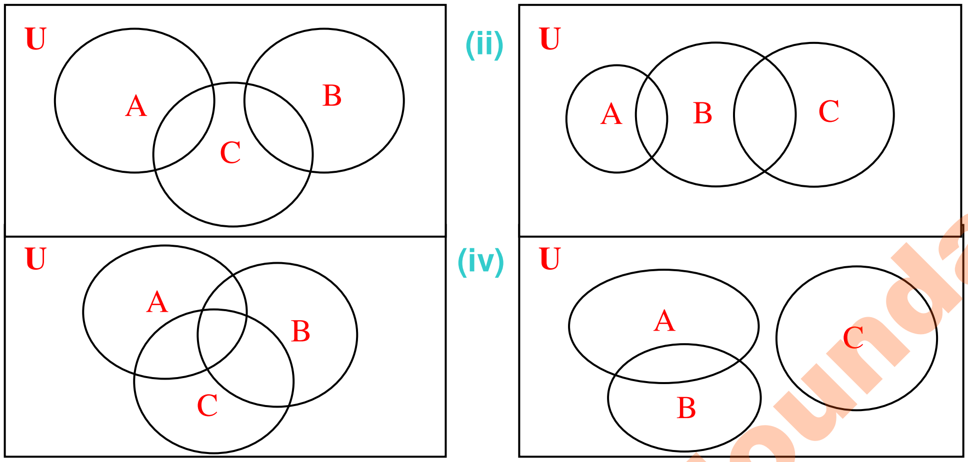
(A ∩ B) ∪ (A ∩ C) =

Exercise 1.4

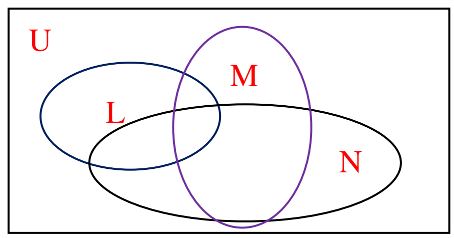
Shade $A \cup (B \cap C)$, $A \cap (B \cup C)$, $(A \cup B) \cup C$ and $A \cap (B \cap C)$

using following Venn diagrams.

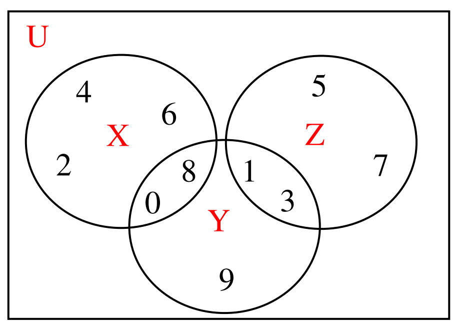
(i) (ii) (iii) (iv)



1. Verify associative law of union and intersection with the help of adjoining diagram.



2. Verify the following properties with the help of adjoining figure.
   (i) distributive property of union over intersection
   (ii) distributive property of intersection over union



3. Prove by using Venn diagram:
   (a) $(P \cup Q) \cup R = P \cup (Q \cup R)$
   (b) $(P \cap Q) \cap R = P \cap (Q \cap R)$

   when
   (i) $P = \{0, 1, 2, 3\}$, $Q = \{2, 3, 4, 5, 6\}$, $R = \{5, 6, 7, 8, 9\}$
   (ii) $P = \{m, n, o, p, q\}$, $Q = \{r, s, t, u\}$, $R = \{t, u, v, w\}$

5. Verify $X \cup (Y \cap Z) = (X \cup Y) \cap (X \cup Z)$
   using Venn diagram for the following sets.
   $X = \{-1, -2, -3\}$, $Y = \{0, 1, 2, 3\}$, $Z = \{0, \pm 1, \pm 2, \pm 3\}$

6. Verify $X \cap (Y \cup Z) = (X \cap Y) \cup (X \cap Z)$
   $X = \{a, e, i, o, u\}$, $Y = \{e, g, n, r, y\}$, $Z = \{a, b, e, g, l, r\}$

Review Exercise 1

1. Encircle the correct answer in the following.

(i) Number of proper subsets possessed by a singleton set is

(a) one (b) two (c) infinite (d) zero

(ii) Any set A can never be a subset of

(a) A′ (b) superset of A (c) universal set (d) finite set

(iii) If $A \cup B = A$ and $A \cap B = B$, then

(a) $A \subset B$ (b) $A \not\supseteq B$ (c) $A \supseteq B$ (d) $A \neq B$

(iv) Every non empty set possesses only one

(a) improper subset (b) proper subset

(c) super set (d) subset

(v) $\phi \cup \phi' =$

(a) ϕ (b) $\phi \cap \phi'$ (c) U (d) U′

(vi) If $A \subset B$, then A – B is equal to

(a) A (b) B (c) ϕ (d) B – A

(vii) If $A - B = B - A = \phi$, then

(a) $A \subseteq B$ (b) $B \subseteq A$ (c) $A = B$ (d) $A \neq B$

(viii) Set of whole numbers and set of negative integers are

(a) disjoint sets (b) finite sets

(c) overlapping sets (d) empty sets

(ix) Set of common elements of A and A′ is

(a) infinite set (b) singleton set (c) universal set (d) null set

(x) If $A' = U$, then

(a) $A = U$ (b) $A = \phi$ (c) $A' = \phi$ (d) $A \neq \phi$

(xi) A – A′ =

(a) ϕ (b) U (c) A′ (d) A

(xii) (U′)′ =

(a) ϕ (b) ϕ' (c) U′ (d) $(\phi')'$

(xiii) Set of Rational numbers between 9 and 10 is

(a) empty set (b) subset (c) infinite set (d) finite set

(xiv) $A\cup(B\cup C)$ is equal to

(a) $A\cup(B\cap C)$ (b) $A\cap(B\cup C)$ (c) $(A\cup B)\cap C$ (d) $(A\cup B)\cup C$

(xv) Set consisting of all the subsets of a given set is called:

(a) universal set (b) subset (c) null set (d) power set

(xvi) If A = {1, 2} then which of the following is true?

(a) $\{2\}\in P(A)$ (b) $\{2\}\subseteq P(A)$ (c) $\{2\}\supseteq P(A)$ (d) $\{2\}\notin P(A)$

(xvii) If $5\in P$ and $P\subseteq Q$, then which of the following is true?

(a) $5\in Q$ (b) $5\notin Q$ (c) $5\subseteq Q$ (d) $5\supseteq Q$

(xviii) If $A\subseteq U$ then $A\cup A'$ is equal to

(a) ϕ (b) U (c) A (d) A′

(xix) If X = {1, 2, 3, 4} then number of subsets of X are

(a) 4 (b) 8 (c) 16 (d) 12

(xx) Power set of empty set is

(a) infinite set (b) empty (c) not possible (d) not empty

(xxi) If S = {0, 1, 2, 3, …,10}, then n(S) is equal to

(a) 10 (b) 5 (c) 11 (d) 12

2. Find the power set of the following sets.

(i) $X = \{\phi, 0, 1\}$ (ii) Y = {{1}, {1, 2}, {1, 2, 3}}

3. Take two overlapping sets and verify commutative law of union and intersection.

4. Take any three sets of your own choice and verify:

   (i) Distributive law of union over intersection.

   (ii) Distributive law of intersection over union.

4. Verify De Morgan's laws when:

   $U = \{0, 1, 2, 3, \ldots, 20\}$, $E = \{2, 3, 5, 7, 11, 13, 17, 19\}$

   $F = \{1, 3, 5, \ldots, 19\}$.

Key Point

- If A and B are two sets then the set A is called a subset of a set B if each element of the set A is contained in the set B. Symbolically we write $A \subseteq B$ and read it as " A is subset of B".
- If each element of the set A is contained in the set B and there is at least one element of B which is not an element of the set A then the set A is called proper subset of the set B.
- If each element of the set A is contained in the set B and there is no element of the set B which is not an element of the set A then the set A is called an improper subset of the set B.
- The power set of any set is the set consisting of all subsets of it. If A is any set then power set of A is denoted by P(A).
- Commutative property of union: $A \cup B = B \cup A$
- Commutative property of intersection: $A \cap B = B \cap A$
- Associative property of union: $(A \cup B) \cup C = A \cup (B \cup C)$
- Associative property of intersection: $(A \cap B) \cap C = A \cap (B \cap C)$
- If A, B and C be any three sets then distributive law of union over intersection is stated as

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

- If A, B and C be any three sets then distributive law of intersection over union is stated as

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

- De Morgan's laws

  (i) $(A \cup B)' = A' \cap B'$

  (ii) $(A \cap B)' = A' \cup B'$

- Two sets A and B are called overlapping if

  (i) There is at least one element common in both the sets.

  (ii) The set A and the set B are not subsets of each other.

2

REAL NUMBERS

This is 12 days unit (periods including homework)

After studying this unit the students will be able to:

- define irrational numbers and differentiate between rational and irrational numbers.
- recognize terminating and non-terminating decimals.
- find square of a number and stablished pattern of such numbers.
- find square root of a Natural number, a common fraction and a decimal.
- find square root of a number which is not a perfect square.
- solve real life problems involving square roots.
- recognize cubes and perfect cubes.
- find cube root of a number which are perfect cubes.

Reading

2.1 IRRATIONAL NUMBERS

2.1.1 Definition of Irrational Number

The numbers which cannot be written in the form of $\frac{p}{q}$ where p and q are integers and q ≠ 0, are called irrational numbers. For example $\sqrt{2}, \sqrt{7}$ etc.

Some more examples of irrational numbers are $\sqrt{3}, \sqrt{5}, \sqrt{12}, \sqrt{66}$ ….

The set of irrational numbers is generally denoted by I.

2.1.2 Recognition of Rational and Irrational Numbers

We know that a number which can be expressed in the form of $\frac{p}{q}$ where p and q are integers and q is not zero is called a rational number.

For example $\frac{2}{3}, -\frac{1}{5}, 3\frac{5}{8}$ and 0.5 etc. are rational numbers.

These numbers can be expressed as ratios of two integers. That is why they are called rational numbers.

As there is no such rational number whose square is 3. Therefore, we can say that $\sqrt{3}$ is not a rational number.

Similarly, $\sqrt{2}, \sqrt{7}, \sqrt{11}, \frac{\sqrt{5}}{3}, \frac{7}{\sqrt{2}}, \pi$ etc. are not rational numbers. These numbers cannot be expressed as ratios of two integers. Such numbers are called irrational numbers.

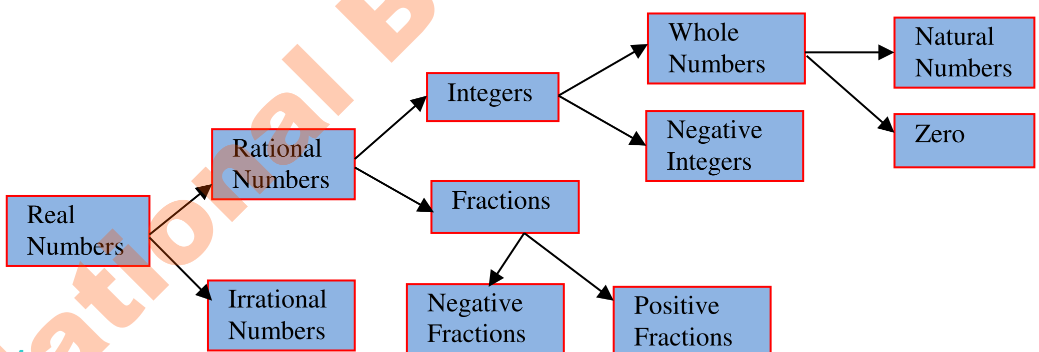
2.1.3 Real Numbers

A number which is either a rational or an irrational number is called a real number. The set of real numbers include all the rational numbers and all the irrational numbers. For example 6, −2, 0.34, $\sqrt{25}$, $\frac{3}{7}$, $\sqrt{10}$, $\sqrt{17}$ and π are real numbers.

If Q represents the set of rational numbers and I represents the set of irrational numbers, then R = Q $\cup$ I.

Set of rational numbers (Q) and set of irrational numbers (I) are disjoint sets and their union is set of real numbers (R). We can write, R = Q $\cap$ I = Φ

We can represent the numbers in the following tree diagram.



Example 1:

Which of the following are rational and irrational numbers?

$$\sqrt{2}, \frac{4}{7}, \sqrt{5}, \sqrt{4}, \sqrt{7}, \frac{\sqrt{16}}{9}, \sqrt{15}, \sqrt{19}, \frac{11}{21},$$

Solution:

Rational numbers: $\frac{4}{7}$, $\sqrt{4} = 2$, $\frac{\sqrt{16}}{9} = \frac{4}{9}$, $\frac{11}{20}$

Irrational numbers: $\sqrt{2}, \sqrt{5}, \sqrt{7}, \sqrt{15}, \sqrt{19}$

2.1.4 Demonstration of non Terminating non Repeating Decimals

Rational and irrational numbers can also be expressed in decimal form but first we recall the types of decimal fractions. There are two kinds of decimal fractions.

(i) Terminating Decimal

The decimals in which there are finite number of digits in its decimal part is called terminating decimals. For example, 2, 0.12, 0.125 etc.

Decimal representation of any real number is either terminating or non terminating

Example 2:

Write the decimal representation of $\frac{37}{100}$.

Solution:

$\frac{37}{100}$ = 0.37, which is a terminating decimal.

All terminating decimals are rational numbers.

Since 37 and 100 are integers, so 0.37 is a rational number.

(ii) Non Terminating Decimal

If there is infinite number of digits after decimal point (on the right of decimal point), the decimal is called non terminating decimal.

e.g. 0.44444… , 3.141592653589… , 2.53333…

Non–terminating decimals are of two types.

(a) Repeating decimals (b) Non-repeating decimals

(a) Repeating Decimals

A non–terminating decimal in which a single digit or a block (group) of digits repeats itself infinite number of times after decimal point, is called recurring decimal. e.g.

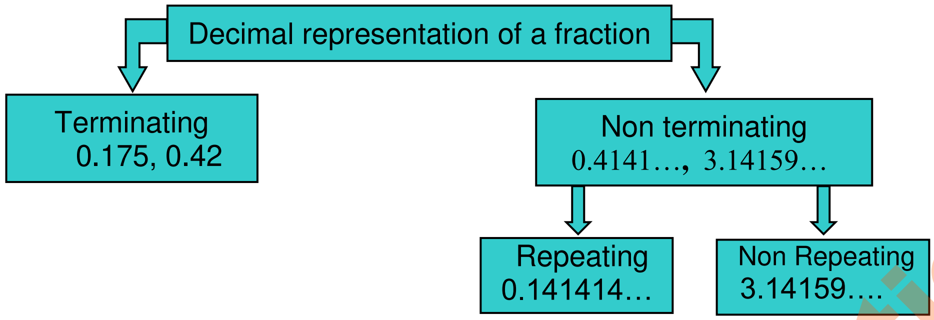
$-\frac{2}{7}$ = − 0.285714285714285714…, $\frac{1}{11}$ = 0.09090909…

(b) Non–repeating Decimals

A non terminating decimal fraction in which digits after decimal point do not repeat in any order. e.g.

All non terminating non recurring decimals are irrational numbers.

1.7321538…, 2.739975947… and 3.1415926535897…etc.



History a Mystery

Babylonians (2000 B.C.) considered the value of $\pi = 3$ and Egyptians (1700 B.C.) considered $\pi = \left(\frac{16}{9}\right)^2$.

Exercise 2.1

1. Which of the following are rational and which are irrational numbers?

(i) $\sqrt{9}$ (ii) 12 (iii) $\frac{5}{9}$ (iv) $\sqrt{8}$ (v) $\sqrt{100}$

(vi) $\frac{13}{2}$ (vii) $\sqrt{126}$ (viii) $\frac{25}{9}$ (ix) $\sqrt{169}$ (x) $\sqrt{26}$

2. Write the following in decimal representation and state which of them are terminating and non-terminating decimals.

(i) $\frac{4}{9}$ (ii) $\frac{13}{20}$ (iii) $\frac{1}{6}$ (iv) $\frac{7}{3}$ (v) $\frac{9}{8}$

(vi) $\frac{13}{8}$ (vii) $\frac{11}{15}$ (viii) $\frac{7}{11}$

3. Name the type of following decimals. Also state whether they are rational or irrational numbers?

(i) 1.2578 (ii) 0.333333… (iii) 1.4142135662…

(iv) 5.1428557142857… (v) 2.236067977…

(vi) 4.36363636… (vii) 4.123105626…

Reading

2.2 PERFECT SQUARE

(a) Square of a Number

When a number is multiplied by itself, the product is called square of that number, e. g.

5×5 is the square of 5 i.e. $5 \times 5 = 5^2 = 25$

The number 25 is called a perfect square.

Similarly

$$6 \times 6 = 6^2 = 36 \text{ (Square of 6 is 36)}$$
$$11 \times 11 = 11^2 = 121 \text{ (Square of 11 is 121)}$$
$$20 \times 20 = 20^2 = 400 \text{ (Square of 20 is 400)}$$
$$23 \times 23 = 23^2 = 529 \text{ (Square of 23 is 529)}$$

The numbers 36, 121, 400 and 529 are called **perfect squares,** because these can be written in the form of squares like

$36 = 6^2$, $121 = 11^2$, $400 = 20^2$, $529 = 23^2$.

The numbers which cannot be represented in the form of squares are not perfect squares. For example the numbers 7, 13, 18, 22, 65, 500, 901 and 1230 etc. are not perfect squares.

(b) Pattern of a Square of Numbers

We can establish a pattern from the square of a numbers. For example

$$1^2 = 1$$
$$2^2 = 4 = 1+2+1$$
$$3^2 = 9 = 1+2+3+2+1$$
$$4^2 = 16 = 1+2+3+4+3+2+1$$
$$5^2 = 25 = 1+2+3+4+5+4+3+2+1$$

....

$$11^2 = 121=1+2+3+4+5+6+7+8+9+10+11+10+9+8+7+6+5+4+3+2+1$$

Similarly,

$16^2 = 256 = 1+2+3+4+5+6+7+8+9+10+11+12+13+14+15+16+15+14+13+12+11+10+9+8+7+6+5+4+3+2+1$

We can also write the patterns in this way

1	$1^2 = 1$
1+2+1	$2^2 = 4$
1+2+3+2+1	$3^2 = 9$
1+2+3+4+3+2+1	$4^2 = 16$
1+2+3+4+5+4+3+2+1	$5^2 = 25$
1+2+3+4+5+6+5+4+3+2+1	$6^2 = 36$
1+2+3+4+5+6+7+6+5+4+3+2+1	$7^2 = 49$
....	....
....	....

Exercise 2.2

1. Find the squares of the following numbers.

   (i) 8 (ii) 12 (iii) 17 (iv) 25 (v) 39
   (vi) 100 (vii) 125 (viii) 200 (ix) 500 (x) 900

2. Tell which of the following are perfect squares.

   (i) 64 (ii) 82 (iii) 99 (iv) 144 (v) 900
   (vi) 125 (vii) 169 (viii) 250

3. Write the patterns of the square of the following numbers.

   (i) $3^2 = 9 = 1 + 2 + 3 + 2 + 1$ (like this pattern)
   (ii) 8^2 (iii) 9^2 (iv) 10^2 (v) 12^2 (vi) 15^2 (vii) 20^2

Reading

2.3 SQUARE ROOTS

2.3.1 (a) Finding Square Root of Natural Numbers which are Perfect Squares

In the previous grade, we have learnt the square root by the method of factorization and division. For refreshing the knowledge and practice, we include more examples and exercises.

(i) Square Root by Factorization

To find the square root of a perfect square by the method of factorization, we convert the number in factors form.

Example 3:

Find the square root of 2025.

5	2025
5	405
3	81
3	27
3	9
	3

Solution:

$$2025 = \underline{5 \times 5} \times \underline{3 \times 3} \times \underline{3 \times 3}$$

$$= 5^2 \times 3^2 \times 3^2$$

Taking square root of both sides

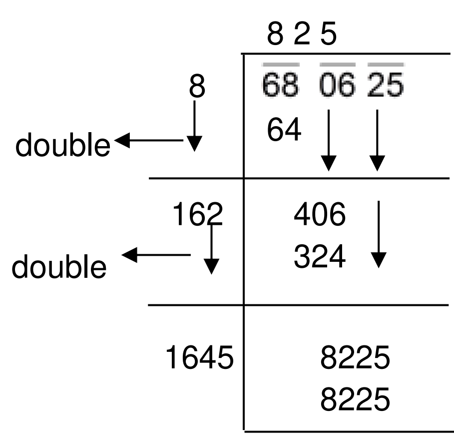
$$\sqrt{2025} = \sqrt{5^2 \times 3^2 \times 3^2}$$

$$= 5 \times 3 \times 3$$

$$= 45$$

(ii) Square Root by Division Method

In Division method, the following steps may be noted.



(a) Place a bar over every pair of digits starting from the unit digit (Right side)

(b) Find the largest number whose square is less than or equal to the number from left first bar. i.e. $(8^2 < 68 < 9^2)$

(c) Bring down the numbers under the next bar to the right of the remainder.

(d) Double the quotient and write it below i.e. (8 + 8 = 16)

(e) Find the number to become the new digit in the quotient.

i.e. $(162 \times 2) < 406 < (163 \times 3)$

(f) Bring down the number under the next bar and find new remainder.

(g) Add the number 2 i.e. 162 + 2 = 164

(h) Find the number again to become the new digit in the quotient.

$$1645 \times 5 = 8225$$

$$\text{So } \sqrt{680625} = 825$$

Example 4:

Find the square roots of

(i) 1296 **(ii)** 355216 by division method.

Solution: (i)

	36
3	$\overline{12}\ \overline{96}$
	9
66	396
	396
	0

$\therefore \sqrt{1296} = 36$

(ii)

	596
5	$\overline{35}\ \overline{52}\ \overline{16}$
	25
109	1052
	981
1186	7116
	7116
	0

$\therefore \sqrt{355216} = 596$

Exercise 2.3

1. Find the square root of the following by factorization.

(i) 256 (ii) 400 (iii) 729 (iv) 1296

(v) 2304 (vi) 20736 (vii) 38416 (viii) 50625

2. Find the square root by division method.

(i) 324 (ii) 4356 (iii) 6561 (iv) 12544

(v) 181476 (vi) 531441

Reading

2.3.1(b) Finding the Square Root of Common Fractions

To find square root of proper and improper fraction, we take square root of numerator and denominator separately. Mixed fraction can also be converted into improper fraction while finding the square root.

Keep in mind the following rules.

(i) $\sqrt{a \times b} = \sqrt{a} \times \sqrt{b}$

(ii) $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$ for $b \neq 0$

The square root of fractional numbers whose numerators and denominators are perfect square.

Square root of common fraction = $\frac{\text{Square root of numerator}}{\text{Square root of denominator}}$

Example 5:

Find $\sqrt{\frac{225}{625}}$

5	225
5	45
3	9
3	3
	1

5	625
5	125
5	25
5	5
	1

Solution:

We know that $\sqrt{\frac{225}{625}} = \frac{\sqrt{225}}{\sqrt{625}}$

$= \frac{\sqrt{15 \times 15}}{\sqrt{25 \times 25}}$

$= \frac{15}{25} = \frac{3}{5}$

Example 6: Simplify $\sqrt{14\frac{1}{16}}$

Solution:

$$\sqrt{14\frac{1}{16}} = \sqrt{\frac{225}{16}} = \frac{\sqrt{3 \times 3 \times 5 \times 5}}{\sqrt{2 \times 2 \times 2 \times 2}} = \frac{\sqrt{3^2 \times 5^2}}{\sqrt{2^2 \times 2^2}} = \frac{3 \times 5}{2 \times 2} = \frac{15}{4} = 3\frac{3}{4}$$

Exercise 2.4

Find the square root of the following.

1. $\frac{25}{49}$ 2. $\frac{225}{169}$ 3. $\frac{1681}{841}$ 4. $\frac{361}{625}$

5. $\frac{1296}{1225}$ 6. $\frac{3025}{729}$ 7. $\frac{2116}{2601}$ 8. $\frac{2025}{1444}$

Simplify the following.

9. $\sqrt{4\frac{29}{49}}$ 10. $\sqrt{10\frac{6}{25}}$ 11. $\sqrt{9\frac{67}{121}}$ 12. $\sqrt{7\frac{21}{25}}$

Reading

2.3.1(c) Square Root of Decimal Numbers

Square root of decimal number without converting it into a rational number, we have to remember the following steps.

(i) Place bar on the integral part of the number from right to left.

(ii) Place bar on the decimal part beginning with the decimal place from left to right and we may place zero at the end to make a pair.

(iii) Find the square root by division method as usual.

(iv) Place decimal point in the quotient as soon as the integral part is ended.

(v) When the remainder is zero, the quotient at this stage is a square root.

Example 7:

Find the square root of 57.76.

	7.6
7	$\overline{57}.\overline{76}$
	49
146	876
	876
	0

Solution:

$\sqrt{57.76} = 7.6$

Example 8:

Find the square root of 0.004096.

Solution:

	0.064
0	$0.\overline{00}\ \overline{40}\ \overline{96}$
	00
6	40
	36
124	496
	496
	0

$\therefore \sqrt{0.004096} = 0.064$

> **Irrational Number**
> A number whose decimal is non–terminating and non–recurring.
> i.e. $\sqrt{3}, \sqrt{5}$ etc.

Exercise 2.5

Find the square root of the following.

1.	0.16	**2.**	20.25	**3.**	46.24	**4.**	0.1296
5.	9.8596	**6.**	42.5104	**7.**	0.000225	**8.**	727.9204
9.	207.0721	**10.**	460.1025				

Reading

2.3.2 Square Root of a Number which is not a Perfect Square

To find square root of a number which is not a perfect square, we may add a suitable number of zeros to the right of the number before applying this method. This method is explained with the help of following examples.

Example 9:

Find the square root of 2 correct upto three places of decimal.

Solution:

We find a number that is approximately equal to $\sqrt{2}$ and has three decimal places, for this we will write three pairs of zeros on the right of 2.

	1.414
1	$\overline{2}.\overline{00}\ \overline{00}\ \overline{00}$
	1
24	100
	96
281	400
	281
2824	11900
	11296
	604

$\therefore \sqrt{2} = 1.414$

Example 10:

Find the square root of 23.1 up to two places of decimal.

Solution:

To find $\sqrt{23.1}$ up to two place of decimal, we write three zeros to the right of 1 to make two pairs of numbers after decimal point.

	4.80
4	$\overline{23}.\overline{10}\ \overline{00}$
	16
88	710
	704
960	600
	000
	600

$\therefore \sqrt{2.31} = 4.80$

Exercise 2.6

Find the square root of the following number up to three places of decimal.

1. 3 **2.** 5 **3.** 7 **4.** 2.5 **5.** 13

6. 1.1 **7.** 20 **8.** 1.7 **9.** 0.9 **10.** $2\frac{1}{12}$

Reading

2.3.3 To Determine the number of Digits in the Square Root of a Perfect Square

To find the number of digits in the square root of a number the following rule may be remembered.

Rule I: Let n be the number of digits in the perfect square then its square root contains $\frac{n}{2}$ digits if n is even.

For example, the number of digits in the perfect square 246016 are 6 which is an even number. According to Rule I, its square root must contain $\frac{6}{2}=3$ digits. Now the square root of 246016 is 496, which are 3 digits.

Similarly 36 is square roots of 1296. 1296 is a 4-digit number and its square root 36 consists of $\frac{4}{2}=2$ digits.

Rule II: Let n be the number of digits in the perfect square then its square root contains $\frac{n+1}{2}$ digits if n is odd.

For example, the number of digits in the perfect square 121 are 3 which is an odd number. According to Rule II, its square root must contain $\frac{3+1}{2}=2$ digits. Now the square root of 121 is 11, which are 2 digits.

Similarly 123 is square roots of 15129. 15129 is a 5-digit number and its square root 123 consists of $\frac{5+1}{2}=3$ digits.

2.3.4 Real Life Problems Involving Square Root

We can solve many problems related to daily life by applying of square root.

Example 11:

The area of a square field is 144.9616 sqm. Find the length of each side.

Solution:

To find length of one side we will find the square root of 144.9616.

12.04

1	$1\,\overline{44}.\overline{96}\,\overline{16}$
	1
22	44
	44
240	96
	00
2404	9616
	9616
	0

Length of one side = 12.04 m

Example 12:

Some students of grade VIII contributed as many rupees as the number of students. If the total collection was Rs.27225, find the number of the students and the amount contributed by each.

Solution:

As the number of students is equal to the number of rupees contributed by each. So the total amount would be equal to the product of two equal factors.
To find the number of students as well as the amount contributed by each, we have to find square root of 27225.

	165
1	$2\,\overline{72}\,\overline{25}$
	1
26	172
	156
325	1625
	1625
	0

So each student contributed = Rs.165
and number of students = 165.

Example 13:

The area of a rectangular garden is $84500m^2$. The length of garden is 5 times its width. Find length and width of the garden.

Solution:

Let the width of garden be y m, then the length will be 5y m.

According to the given condition:

$$y(5y) = 84500$$
$$5y^2 = 84500$$
$$y^2 = 16900$$

Taking square root of both the sides

$$\sqrt{y^2} = \sqrt{16900}$$
$$y = 130$$

So width = 130m

Length = 5 (130)

= 650m

Exercise 2.7

1. The area of a square public park is 19600 square metres. Find the length of the side of the park.
2. Area of a circular field is $2464m^2$. Find the circumference of the circle. (Take $\pi \approx \frac{22}{7}$)
3. The students of a school contributed as many rupees as the number of students for a picnic. If the total collection was Rs.1449616, then find the number of the students and the amount contributed by each.
4. The area of a square shaped hall is $225m^2$. Find its perimeter.
5. Find the least number which must be subtracted from 3151 to make it a perfect square.
6. The area of a rectangular field is $230496cm^2$. The length of field is 6 times its width. Find the length and width of field.

7. The length of a rectangular plot is $2\frac{1}{2}$ times of width. If the area of rectangular plot is 12250m^2, find its length and width.

8. The area of a square lawn of a school is 42025m^2. If you complete 5 rounds of the square lawn. How much distance you traveled?

Reading

2.4 CUBE AND CUBE ROOTS

2.4.1 Perfect Cubes

In mathematics cube mean the multiplication of a number three times.

For example

$2 \times 2 \times 2 = 2^3 = 8$

$3.5 \times 3.5 \times 3.5 = 3.5^3 = 42.875$

Similarly

$$7 \times 7 \times 7 = 7^3 = 343$$

$$\frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} = \left(\frac{2}{3}\right)^3 = \frac{8}{27}$$

Here $2 \times 2 \times 2 = 2^3$ and 2^3 is read as 2 cube.

Similarly 5^3 is read as 5 cube.

Some more examples are

$$64 = 4 \times 4 \times 4 = 4^3$$

$$729 = 9 \times 9 \times 9 = 9^3$$

$$512 = 8 \times 8 \times 8 = 8^3$$

Note that 64, 729, 512 are cubes of whole numbers. These numbers are called perfect cubes. Similarly 8, 27, 125, 216 are perfect cubes.

Now 12 = 2 × 2 × 3. i.e. 12 cannot be written in the form of cube, therefore we can say that 12 is not a perfect cubes. Similarly 9, 15, 48 are not perfect cubes.

The cube root of a number is the inverse of the cube of a number. For example

$$2 \times 2 \times 2 = 2^3 = 8 \quad \text{and} \quad \sqrt[3]{8} = \sqrt[3]{2^3} = 2$$

$$3 \times 3 \times 3 = 3^3 = 27 \quad \text{and} \quad \sqrt[3]{27} = \sqrt[3]{3^3} = 3$$

$$4 \times 4 \times 4 = 4^3 = 64 \quad \text{and} \quad \sqrt[3]{64} = \sqrt[3]{4^3} = 4$$

We observe that cube root of 8 is 2, 27 is 3 and that of 64 is 4.

Example 14:

Find the cube roots of (i) 512 (ii) 2744

(i) 512

$$512 = 2\times2\times2\times2\times2\times2\times2\times2\times2$$
$$= 8 \times 8 \times 8$$
$$= 8^3$$
$$\sqrt[3]{512} = \sqrt[3]{8^3} = 8$$

2	512
2	256
2	128
2	64
2	32
2	16
2	8
2	4
	2

(ii) 2744

$$2744 = 2\times2\times2\times7\times7\times7$$
$$= (2\times7)\times(2\times7)\times(2\times7)$$
$$= 14 \times 14 \times 14$$
$$= (14)^3$$
$$\sqrt[3]{2744} = \sqrt[3]{14^3} = 14$$

2	2744
2	1372
2	686
7	343
7	49
	7

2.4.2 Properties of Cubes of Numbers

Following are some properties of cubes of natural numbers

(i) Cubes of all even natural numbers are even. e.g. $2^3 = 8$

(ii) Cubes of all odd natural numbers are odd. e.g. $3^3 = 27$

(iii) Cubes of numbers ending in digits 1, 4, 5, and 9 are the numbers ending in the same digit.
e.g. $11^3 = 1331$, $14^3 = 2744$, $15^3 = 3375$ etc.

(iv) Cubes of numbers ending in digits 2 ends in digit 8. e.g. $12^3 = 1728$

(v) Cubes of numbers ending in digits 8 ends in digit 2. e.g. $8^3 = 512$.

(vi) Cubes of numbers ending in digits 3 ends in digit 7. e.g. $13^3 = 2197$.

(vii) Cubes of numbers ending in digits 7 ends in digit 3. e.g. $7^3 = 343$.

(viii) Cubes of numbers ending in 6 also end in 6. e.g. $16^3 = 4096$.

(ix) Cubes of all numbers ending in a zero will also end in a zero.

Cubes of natural numbers form an interesting pattern given below.

$1^3 = 1$

$2^3 = 8 = 3 + 5$

$3^3 = 27 = 7 + 9 + 11$

$4^3 = 64 = 13 + 15 + 17 + 19$

$5^3 = 125 = 21 + 23 + 25 + 27 + 29$

$6^3 = 216 = 31 + 33 + 35 + 37 + 39 + 41$

Exercise 2.8

1. Find the cube of the following numbers.

   6, 9, 11, 13, 15, 16, 20, 25

2. Which of the following are perfect cubes?

   21, 27, 48, 64, 125, 216, 300, 729

3. Find the cube root of the following.

   (i) 1331 (ii) 2197 (iii) 4096 (iv) 5832

4. Given that volume of cube is 64 m^3. Find the length of its sides.
5. What is the volume of a cube having side 12cm?

Review Exercise 2

1. Choose correct answer from given choices.

   (i) $\frac{\sqrt{6}}{\sqrt{3}}$ is an

   (a) even number (b) odd number
   (c) irrational number (d) rational number

   (ii) The square root of 144 has …… digits.

   (a) One (b) two (c) three (d) four

   (iii) $\sqrt{1\frac{11}{25}}$ is equal to

   (a) 1.2 (b) 1.25 (c) 0.2 (d) 1.22

(iv) $\frac{\sqrt{7.29}}{2}$ is equal to

(a) 1.33 (b) 2.7 (c) 1.909 (d) 1.35

(v) $\frac{\sqrt{125 \div 25 \times 5}}{\sqrt{125 \div 5}}$ is equal to

(a) 10 (b) 25 (c) 5 (d) 1

(vi) The area of the square region is 196m^2, the length of each side of the region is

(a) 12m (b) 14m (c) 16m (d) 18m

(vii) $\frac{2}{\sqrt{4}} \times \sqrt{\frac{16}{9}}$ is equal to

(a) $\frac{3}{4}$ (b) $\frac{1}{3}$ (c) $\frac{4}{3}$ (d) $\frac{8}{3}$

(viii) A square field has an area of 144 m^2, its perimeter is

(a) 12m (b) 24m (c) 36m (d) 48m

(ix) $\sqrt{1.21}$ is equal to

(a) 1.1 (b) 11 (c) 0.11 (d) 11.1

(x) What is number of digits in square root of a perfect square having 6 digits?

(a) 6 (b) 5 (c) 3 (d) 2

(xi) What is number of digits in square root of a perfect square having7 digits?

(a) 3 (b) 4 (c) 5 (d) 6

(xii) Cube of numbers ending in digit 9 is the number ending in

(a) 9 (b) 7 (c) 5 (d) 3

(xiii) Cube of numbers ending in digit 7 is the number ending in

(a) 9 (b) 7 (c) 5 (d) 3

2. Find the square root of the following.

(i) 7260.7441 (ii) 0.00001296

3. Find the cube root of the following.

(i) 4913 (ii) 8000

Key Point

- The number which cannot be written in the form $\frac{p}{q}$, where $p, q \in Z$ and $q \neq 0$ are called irrational numbers.
- Then union of sets of rational and irrational number is called the set of real numbers and is denoted by R.
- The decimals in which there are finite number of digits in its decimal part is called terminating decimal.
- If there is infinite number of digits after decimal point (on the right of decimal point), the decimal is called non terminating decimal.
- All terminating decimals are rational.
- All non terminating non recurring decimals are irrational numbers.
- A perfect square is never negative.
- A square number never ends in 2, 3, 7 or 8.
- To find square root of a number which is not a perfect square, we may add a suitable number of zeros to the right of the number.
- To find square root of proper and improper fraction, we take square root of numerator and denominator separately.
- If n be the number of digits in the perfect square then its square root contains $\frac{n}{2}$ digits if n is even.
- If n be the number of digits in the perfect square then its square root contains $\frac{n+1}{2}$ digits if n is odd.
- Cubes of all even natural numbers are even and cubes of all odd natural numbers are odd.
- Cubes of numbers ending in digits 0, 1, 4, 5, 6 and 9 are the numbers ending in the same digit.
- Cubes of numbers ending in digits 2 ends in digit 8 and vice versa.
- Cubes of numbers ending in digits 3 ends in digit 7 and vice versa.

3

NUMBER SYSTEM

This is 20 days unit
(periods including homework)
After studying this unit, you will be able to:

- ❖ recognize base of a number system.
- ❖ define number system with base 2, 5, 8 and 10.
- ❖ explain
  - binary number system (system with base 2).
  - number system with base 5.
  - octal number system (system with base 8).
  - decimal number system (system with base 10).
- ❖ convert a number from decimal system to the system with base 2, 5 and 8 and vice versa.
- ❖ add, subtract and multiply numbers with base 2, 5 and 8.
- ❖ add, subtract and multiply numbers with different bases.

Reading

3.1 NUMBER SYSTEM

3.1.1 Concept of a Base of a Number System

(i) Decimal System

Now a days in most of the countries decimal system is used. In decimal system, we use ten digits which are 0, 1, 2, 3, 4, 5, 6, 7, 8, 9.

We know that numbers are combination of digits. Also in a given number, every digit has certain place value. In the base 10 system, the rightmost digit of a natural number has a place value of 1, and every other digit has a place value ten times that of the place value of the digit to its right

For example 32 is not 3 + 2 but 30 + 2 or $3 \times 10 + 2 \times 1$.
It means that place value of 2 is one and that of 3 is ten.

Thus 32 $= (3 \times 10) + (2 \times 1)$

$= 3 \times 10^1 + 2 \times 10^0$

$10^0 = 1$

Similarly 567 = 500 + 60 + 7

$= (5 \times 100) + (6 \times 10) + (7 \times 1)$

$= 5 \times 10^2 + 6 \times 10^1 + 7 \times 10^o$

Here place value of 5 is hundred, place value of 6 is ten and that of 7 is one. We read 567 as 5 hundreds 6 tens and 7 units or five hundred and sixty seven.

Similarly $1023 = (1\times10^3) + (0\times10^2) + (2\times10^1) + (3\times10^o)$

We have noticed that in decimal system, all the place values of digits are in terms of powers of 10. For this reason decimal system is also known as system of base 10. This system is also called denry system. In decimal system we count in bundles of 10.

Example 1:

Find the place value of each digit in $(2734)_{10}$.

Solution:

Digit	2	7	3	4
Place value	thousand	hundred	ten	unit

(ii) Number System with Base Two

The number system with base 2 is also known as binary system. The digits used in this system are 0 and 1. In binary system we count in bundles of two. Place values of digits in the binary system are in terms of powers of 2.

To understand binary system, we perform an activity. Suppose we have fifteen balls. We want to put them into two boxes in such a way that one box contains 10 balls and remaining balls are in the second box as shown in Fig.3.1.

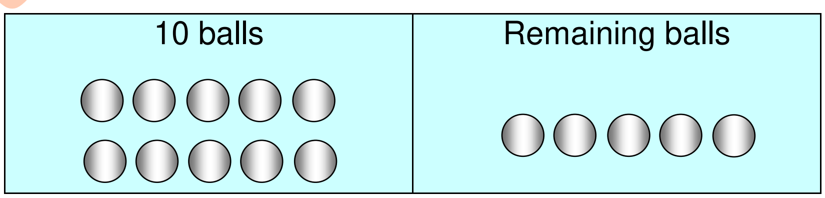


Fig. (3.1)

Here 15 = 10 + 5

$= (1 \times 10^1) + (5 \times 10^o)$

$= (15)_{10}$

Suppose we want to place these balls in four boxes as shown in fig. (3.2)

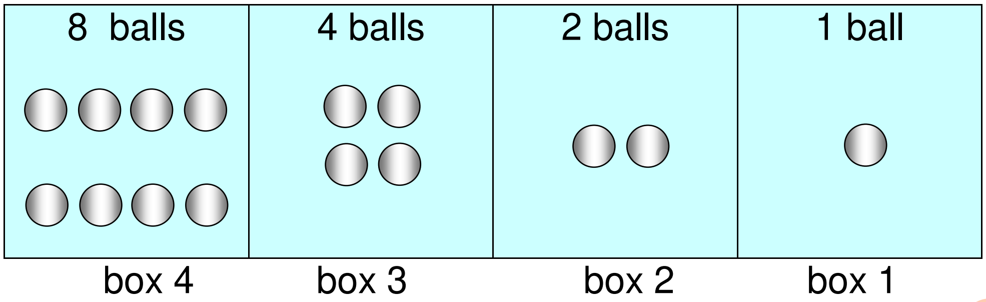


Fig. (3.2)

In box 1 there is 1 ball or 2^0 ball

In box 2 there are 2 balls or 2^1 balls

In box 3 there are 4 balls or 2^2 balls

In box 4 there are 8 balls or 2^3 balls

So Fig.(3.2) represents a new number system in which place values are in terms of power of 2. We can say that counting in this system is in bundles of two. Such a system is known as system of base 2 or binary system. The digits used in this system are 0 and 1.

Now 15 $= 8 + 4 + 2 + 1$

$= (1 \times 2^3) + (1 \times 2^2) + (1 \times 2^1) + 1 \times 2^0$

$= (1111)_2$

i.e. 15 in decimal system = 1111 in binary system.

To avoid confusion among numbers of different systems, we write base of that number a little below.

$\therefore \quad (15)_{10} = (1111)_2$

$(1111)_2$ is read as 1 eight, 1 four, 1 two and 1 unit or one, one, one, one base two.

Thus $(1111)_2 = (1 \times 8) + (1 \times 4) + (1 \times 2) + (1 \times 1)$

$= (1 \times 2^3) + (1 \times 2^2) + (1 \times 2^1) + (1 \times 2^0)$

Example 2:

Find the place value of each digit in the following numbers.

(i) $(10011)_2$

(ii) $(1110010)_2$

Solution:

(i)

Digit	1	0	0	1	1
Place value	sixteen	eight	four	two	unit

(ii)

Digit	1	1	1	0	0	1	0
Place value	sixty four	thirty two	sixteen	eight	four	two	unit

Example 3:

Write the following numbers in words.

(i) $(101010)_2$ **(ii)** $(1000111)_2$

Solution:

(i) $(101010)_2$

1 thirty two, 0 sixteen, 1 eight, 0 four, 1 two and 0 unit or 1 thirty two, 1eight, 1 two

(ii) $(1000111)_2$

1 sixty four, 0 thirty two, 0 sixteen, 0 eight, 1 four, 1 two and 1 unit.

or 1 sixty four, 1 four, 1 two and 1 unit.

Example 4:

Write the following numbers in digits.

(i) 1 sixteen, 1 two and 1 unit.

(ii) 1 thirty two, 1 sixteen, 1 eight and 1 two

Solution:

(i) $(1011)_2$

(ii) $(111010)_2$

(iii) Number System with Base Five

In the base five system we count in the bundles of 5. The digits used in base five system are 0, 1, 2, 3, 4. Place values of digits in base five system are in terms of powers of 5.

Significance of the Binary System

The importance of binary system is due to the fact that it is used in digital computers, calculators, digital watches and digital balances etc. The two digits 0 and 1 are called bits. Eight bits form one byte and 1024 bytes form one kilobyte etc. These are called memory units and are stored in memory storage devices such as floppy, disc etc.

Suppose we put 31 balls in the three boxes as shown in the Fig. (3.3).

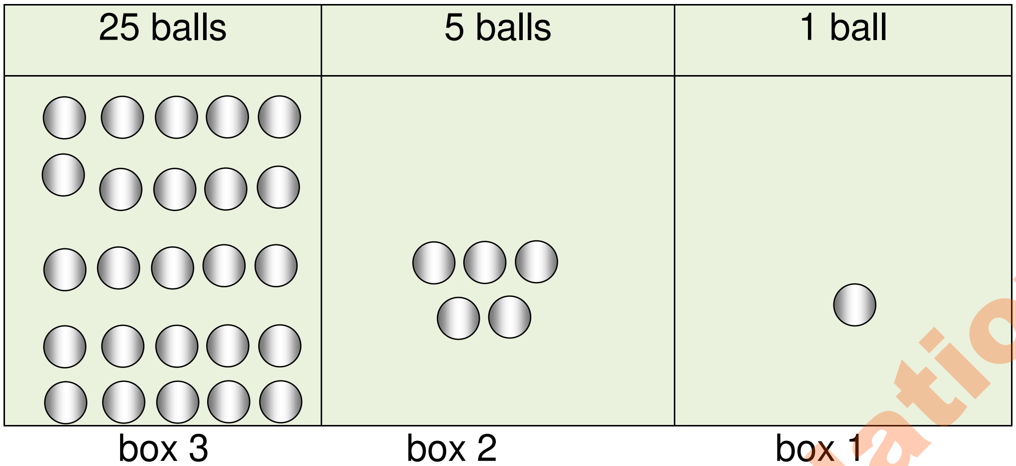


Fig. (3.3)

Thus $31 = 25 + 5 + 1$

$= (1 \times 5^2) + (1 \times 5) + (1 \times 5^o)$

∴ 31 in decimal system = 111 in base five system

i.e. $(31)_{10} = (111)_5$

$(111)_5$ is read as 1 twenty five, 1 five, 1 unit or one, one, one base five.

Digit	1	1	1
Place value	twenty five	five	unit

Similarly $36 = 25 + 10 + 1$

$= (1 \times 5^2) + (2 \times 5^1) + (1 \times 5^o)$

$= (121)_5$

$(121)_5$ is read as 1 twenty five, 2 fives, 1 unit

Example 5:

Find place value of each digit in $(324)_5$.

Solution:

$(324)_5 = (3 \times 5^2) + (2 \times 5^1) + (4 \times 5^o)$

$= (3 \times 25) + (2 \times 5) + (4 \times 1)$

Digit	3	2	4
Place Value	twenty five	five	unit

Example 6:

Write in words the following numbers.

(i) $(103)_5$ (ii) $(2340)_5$

Solution:

(i) 1 twenty five, 3 units or 1 twenty five, 0 five, 3 units

(ii) 2 one hundred twenty fives, 3 twenty fives, 4 fives, 0 unit

Example 7: Write the following in digits.

(i) 3 twenty fives, 1 unit

(ii) 4 one twenty fives, 2 fives, 3 units.

Solution:

(i) $(301)_5$ (ii) $(4023)_5$

(iii) Number System with Base 8

In base eight system we count in bundles of eight. The digits used in this system are 0, 1, 2, 3, 4, 5, 6 and 7. In base eight system we count in bundles of eights. Place values of digits in base eight system are in terms of powers of 8. This system is also known as octal number system.

To understand this system, we perform an activity. Suppose we have seventy three stars. We want to put them into three boxes in such a way that one box contains 64 stars, second box contains 8 stars and 1ball in the third box as shown in Fig.3.4.

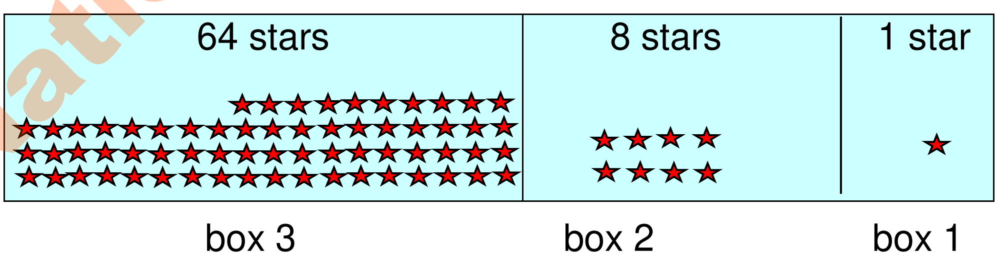


Fig. (3.4)

In box 1 there is 1 star or 2^o star.

In box 2 there are 8 balls or 8^1 stars.

In box 3 there are 64 balls or 8^2 stars.

So Fig.(3.4) represents a new number system in which place values are in terms of power of 8. We can say that counting in this system is in bundles of eight. Such a system is known as system of base 8. The digits used in this system are 0, 1, 2, 3, 4, 5, 6, 7.

Now 73 $= 64 + 8 + 1$

$= (1 \times 64) + (1 \times 8) + (1 \times 1)$

$= (1 \times 8^2) + (1 \times 8^1) + (1 \times 8^o)$

$= 111$

i.e. 73 in decimal system = 111 in base eight system.

$\therefore \quad (73)_{10} = (111)_8$

$(111)_8$ is read as 1 sixty four, 1 eight and 1 unit

or one, one, one base eight.

Digit	1	1	1
Place value	sixty four	eight	unit

Similarly 657 $= 512 + 128 + 16 + 1$

$= (1 \times 8^3) + (2 \times 8^2) + (2 \times 8^1 + (1 \times 8^o)$

$= (1221)_8$

$(1221)_8$ is read as 1 five hundred twelve, 2 sixty fours, 2 eights and 1 unit or one, two, two, one base eight.

Example 8:

Find place value of each digit in $(513)_8$.

Solution:

$(513)_8 = (5 \times 8^2) + (1 \times 8^1) + (3 \times 8^o)$

$= (1 \times 64) + (1 \times 8) + (3 \times 1)$

Digit	1	0	3
Place Value	sixty four	eight	unit

Example 9:

Write in words the following numbers.

(i) $(212)_8$

(ii) $(2340)_8$

Solution:

(i) $(212)_8 = (2 \times 8^2) + (1 \times 8^1) + (2 \times 8^0)$

$= (2 \times 64) + (1 \times 8) + (2 \times 1)$

So $(212)_8$ is read as "2 sixty fours, 1 eight, 2 units".

(ii) 2 five hundred twelve, 3 sixty fours, 4 eights, 0 unit

Example 10: Write the following in digits.

(i) 7 sixty fours, 5 eight, 3 units

(ii) 5 five hundred twelve, 6 eights, 1 unit

Solution:

(i) $(753)_8$

(ii) $(5061)_8$

Exercise 3.1

1. Write the following binary numbers in words.

(i) $(1011)_2$ **(ii)** $(11110)_2$ **(iii)** $(100011)_2$ **(iv)** $(1100110)_2$

2. Write the following numbers in binary digits.

(i) 1 sixteen, 0 eight, 0 four, 1 two, 1 unit

(ii) 1 thirty two, 1 eight, 1 four, 1 two

(iii) 1 sixty four, 1 thirty two, 1 sixteen, 1 unit

3. Write the following base five numbers in words.

(i) $(124)_5$ **(ii)** $(2233)_5$ **(iii)** $(10411)_5$ **(iv)** $(4003)_5$

4. Write the following numbers in base five digits.

(i) 3 twenty five, 2 fives, 1 unit

(ii) 1 one hundred twenty five, 4 twenty fives, 3 fives

(iii) 2 six hundred twenty fives, 1 twenty five, 2 fives, 1 unit

5. Write the following octal numbers in words.

(i) $(34)_8$ **(ii)** $(650)_8$ **(iii)** $(1155)_8$ **(iv)** $(6002)_8$

6. Write the following numbers in base eight digits.

(i) 5 sixty fours, 4 unit

(ii) 6 five hundred twelve, 4 sixty fours, 3 eights, 7 units

(iii) 1 five hundred twelve, 5 eights, 6 units

7. Write place values of each digit in the following numbers.

(i) $(1203)_{10}$ (ii) $(52341)_{10}$ (iii) $(10101)_2$
(iv) $(100111)_2$ (v) $(1001101)_2$ (vi) $(4103)_5$
(vii) $(12204)_5$ (viii) $(40341)_5$ (ix) $(513)_8$
(x) $(6701)_8$ (xi) $(1254)_8$ (xii) $(2043)_8$

Reading

3.2 CONVERSION OF A NUMBER FROM DECIMAL SYSTEM TO THE SYSTEM WITH BASE 2, 5 AND 8 AND VICE VERSA

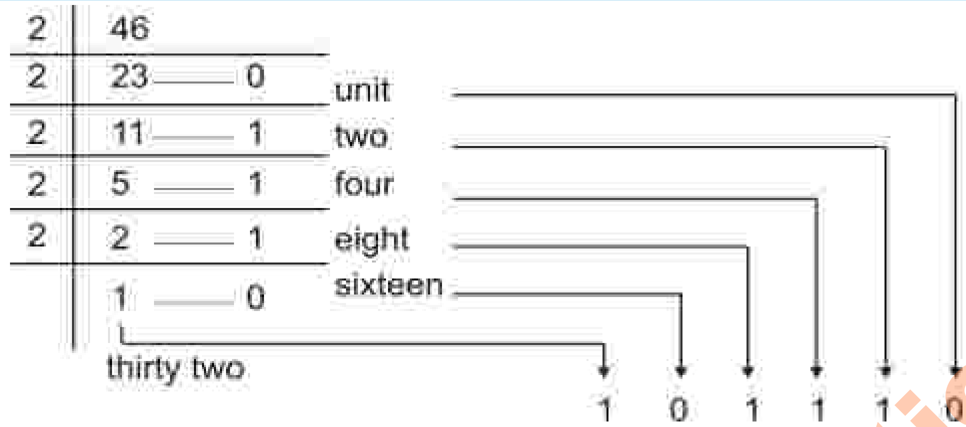
3.2.1 (a) Conversion of Numbers of Decimal System into Binary System

Let us convert 15 into binary system. We know that all the place values in binary system are in terms of powers of 2.

$\therefore$ $15 = 8 + 4 + 2 + 1$
$= (1 \times 2^3) + (1 \times 2^2) + (1 \times 2^1) + (1 \times 2^o)$
$= (1111)_2$

Similarly $46 = 32 + 8 + 4 + 2$
$= 32 + 0 + 8 + 4 + 2 + 0$
$= 1 \times 32 + 0 \times 16 + 1 \times 8 + 1 \times 4 + 1 \times 2 + 0 \times 1$
$= (1 \times 2^5) + (0 \times 2^4) + (1 \times 2^3) + (1 \times 2^2) + (1 \times 2^1) + (0 \times 2^o)$
$= (101110)_2$

But this method is difficult when numbers are of large value. Therefore we explain another method known as method of successive (continuous) division. In this method a number of decimal system is divided by 2 continuously till the remainder becomes less than 2 i.e. 0 or 1. Let us see how



∴ 46 = $(101110)_2$

Example 11:

Convert 350 into binary system.

Solution:

2	350	
2	175	0
2	87	1
2	43	1
2	21	1
2	10	1
2	5	0
2	2	1
	1	0

> **Plus the Memory**
> In decimal system we read 1011 as 1 thousand, 0 hundred, 1 ten and 1 unit while in binary system 1011 is read as 1 eight, 0 four, 1 two and 1 unit.

∴ 350 = $(101011110)_2$

3.2.1 (b) Conversion of Number from Binary System to Decimal System

We illustrate the method of conversion by the following examples.

Example 12:

Convert the following binary numbers into an equivalent number in decimal system.

(i) $(110110)_2$ **(ii)** $(1110110)_2$

Solution:

(i) $(110110)_2$

$= (1 \times 2^5) + (1\times 2^4) + (0 \times 2^3) + (1 \times 2^2) + (1 \times 2^1) + (0 \times 2^o)$

$= (1 \times 32) + (1 \times 16) + (0 \times 8) + (1 \times 4) + (1 \times 2) + (0 \times 1)$

$= 32 + 16 + 0 + 4 + 2 + 0 = 54$

(ii) $(1110110)_2$

$= (1 \times 2^6) + (1\times2^5) + (1\times2^4) + (0\times2^3) + (1\times2^2) + (1\times2^1) + (0\times2^o)$

$= (1\times64) + (1\times32) + (1\times16) + (0\times8) + (1\times 4) + (1\times 2) + (0\times1)$

$= 64 + 32 + 16 + 0 + 4 + 2 + 0 = 118$

3.2.1(c) Conversion of Decimal Numbers to Base Five System

As discussed in binary system, there are the following two methods to convert a number from one number system to another number system.

(i) Method of place value **(ii)** Method of successive division

5^o

Example 13:

Convert 86 to an equivalent number in base five system.

Solution:

First method

$86 = 75 + 10 + 1$

$= (3 \times 25) + (2 \times 5) + (1 \times 1)$

$= (3 \times 5^2) + (2 \times 5^1) + (1 \times 5^o) =$

Second Method

5	86
5	17–1
	3 – 2

$\therefore 86 = (321)_5$

Note: First method used in above example becomes difficult for the numbers of large value. Therefore we prefer second method i.e. method of successive division.

Example 14:

Convert 56721 into an equivalent number with base five.

Solution:

5	56721
5	11344 – 1
5	2268 – 4
5	453 – 3
5	90 – 3
5	18 – 0
	3 – 3

$\therefore$ 56721 = (3303341)

3.2.1 (d) Converting a Number of Base Five System to Decimal System

We illustrate the method with the help of following examples.

Example 15:

Convert (i) $(432)_5$ (ii) $(132402)_5$ into equivalent decimal numbers.

Solution:

(i) $(432)_5$

$= (4 \times 5^2) + (3 \times 5^1) + (2 \times 5^o)$

$= (4 \times 25) + (3 \times 5) + (2 \times 1)$

$= 100 + 15 + 2 = 117$

(ii) $(132402)_5$

$= (1 \times 5^5) + (3 \times 5^4) + (2 \times 5^3) + (4 \times 5^2) + (0 \times 5^1) + (2 \times 5^o)$

$= (1 \times 3125) + (3 \times 625) + (2 \times 125) + (4 \times 25) + (0 \times 5) + (2 \times 1)$

$= 3125 + 1875 + 250 + 100 + 0 + 2$

$= 5352$

3.2.1(e) Conversion of Decimal Numbers to Base Eight System

As discussed above, there are the following two methods to convert a number from one number system to another number system.

(i) Method of place value (ii) Method of successive division

Example 16:

Convert 20 to an equivalent number in base eight system (octal number system).

Solution:

First method

$20 = 16 + 4$

$= (2 \times 8) + (4 \times 1)$

$= (2 \times 8^1) + (4 \times 8^o)$

$= (24)_8$

Second method

8	20
	2 – 4

$\therefore \quad 20 = (24)_5$

Example 17:

Convert 56721 into an equivalent number with octal number system.

Solution:

8	56721
8	7090 – 1
8	886 – 2
8	110 – 6
8	13 – 6
	1 – 5

$\therefore$ $56721 = (156621)_8$

3.2.1 (f) Converting a Number of Base Eight System to Decimal System

We illustrate the method with the help of following examples.

Example 18:

Convert **(i)** $(465)_8$ **(ii)** $(105417)_8$ into equivalent decimal numbers.

Solution:

(i) $(465)_8$

$= (4 \times 8^2) + (6 \times 8^1) + (5 \times 8^o)$

$= (4 \times 64) + (6 \times 8) + (5 \times 1)$

$= 256 + 48 + 5 = 309$

(ii) $(105417)_8$

$= (1 \times 8^5) + (0 \times 8^4) + (5 \times 8^3) + (4 \times 8^2) + (1 \times 8^1) + (7 \times 8^o)$

$= (1 \times 32768) + (0 \times 4096) + (5 \times 512) + (4 \times 64) + (1 \times 8) + (7 \times 1)$

$= 32768 + 0 + 2560 + 256 + 8 + 7 = 35599$

Exercise 3.2

1. Convert the following decimal numbers into an equivalent number with base 2.

(i) 9 **(ii)** 20 **(iii)** 37 **(iv)** 60
(v) 111 **(vi)** 864 **(vii)** 1578 **(viii)** 1300

2. Convert the following numbers of binary system into decimal system.

(i) $(10)_2$ **(ii)** $(111)_2$ **(iii)** $(100101)_2$
(iv) $(1110011)_2$ **(v)** $(1010101001)_2$ **(vi)** $(100110101)_2$

3. Convert the following decimal numbers into equivalent numbers of base five system.

(i)	8	(ii)	18	(iii)	32	(iv)	65
(v)	123	(vi)	306	(vii)	729	(viii)	1999
(ix)	2104	(x)	5000	(xi)	26921	(xii)	60917

4. Convert the following numbers into decimal system.

(i)	$(43)_5$	(ii)	$(214)_5$	(iii)	$(2431)_5$	(iv)	$(31224)_5$
(v)	$(100232)_5$	(vi)	$(203404)_5$	(vii)	$(102030)_5$	(viii)	$(44444)_5$
(ix)	$(112233)_5$						

5. Convert the following decimal numbers into equivalent numbers of base eight system.

(i)	8	(ii)	22	(iii)	37	(iv)	69
(v)	132	(vi)	700	(vii)	624	(viii)	1789
(ix)	2013	(x)	4760	(xi)	27823	(xii)	61092

6. Convert the following octal numbers into decimal system.

(i)	$(63)_8$	(ii)	$(217)_8$	(iii)	$(2435)_8$	(iv)	$(31264)_8$
(v)	$(10237)_8$	(vi)	$(20544)_8$	(vii)	$(100230)_8$	(viii)	$(55555)_8$
(ix)	$(7777)_8$						

Reading

3.2.2 Adding, Subtracting and Multiplying the Numbers with Base 2 (Binary System)

3.2.2 (a) Addition of Numbers in Binary System

We know that there are two digits in binary system i.e. 0 and 1. They can be added in the following four ways.

$0 + 0 = 0$, $0 + 1 = 1$, $1 + 0 = 1$, $1 + 1 = 2 = (10)_2$

While adding the two numbers in binary system when sum of two digits is greater than 1, the resulting sum is converted into an equivalent number in binary system. The unit digit so obtained is placed below and other digit (digits) is added to the digits in the next column. For example we add $(111)_2$ and (110).

$$\begin{array}{r} (1\ 1\ 1)_2 \\ +\ (1\ 1\ 0)_2 \\ \hline (1\ 1\ 0\ 1)_2 \end{array}$$

(i). Start by adding 0 + 1 = 1 in the first column.
(ii). In the second column 1 + 1 = 2 which is 10 in binary system.
(iii). Write 0 below the second column and carry 1 to the third column.
(iv). In the third column 1 + 1 + 1 = 3 which is 11 in binary system.

$$\begin{array}{r} \color{red}{1\ 0\ 1\ 1} \\ (1\ 1\ 1\ 1)_2 \\ (1\ 1\ 0\ 1)_2 \\ +\ (1\ 1\ 1\ 1)_2 \\ \hline (1\ 0\ 1\ 0\ 1\ 1)_2 \end{array}$$

Let us take another example.

(i). Start by adding 1 + 1 + 1 = 3 which is 11 in binary system.
(ii). Write 1 below the first column and carry 1 to the second column.
(iii). In the second column 1 + 1 + 1 = 3 and repeat the same process as in the first column.
(iv). In the third column 1 + 1 + 1 + 1 = 4 which is 100 in binary system.
(v). Write 0 below the third column and carry 10 to the next column.
(vi). Look carefully the position of 10 written above the fourth column. Clearly 1 is gone to the fifth column.
(vii). In the fourth column, add 0 + 1 + 1 + 1 = 3 which is 11 in binary system. Repeat the process of the first column.
(viii). Finally add 1 + 1 = 2 which is 10.

Example 19:

Add the following binary numbers.

(i) $(110)_2$, $(111)_2$ **(ii)** $(1010111)_2$, $(111111)_2$

Solution:

(i)

$$\begin{array}{r} \color{red}{1} \\ (1\ 1\ 0)_2 \\ +\ (1\ 1\ 1)_2 \\ \hline (1\ 1\ 0\ 1)_2 \end{array}$$

(ii)

$$\begin{array}{r} (1\ 0\ 1\ 0\ 1\ 1\ 1)_2 \\ +\ (1\ 1\ 1\ 1\ 1\ 1)_2 \\ \hline (1\ 0\ 0\ 1\ 0\ 1\ 1\ 0)_2 \end{array}$$

3.2.2(b) Subtraction of Numbers in Binary System

Two digits 0, 1 are subtracted in the following way.

0 – 0 = 0 , 1 – 1 = 0 , 1 – 0 = 1

Now since 1 + 1= 10 in binary system.

$\therefore$ 10 – 1 = 1

While subtracting two numbers in binary system when a larger number is subtracted from smaller number, we borrow 1 from next column which becomes 2 in the previous column.

Let us take the following examples.

Example 20:

Subtract $(111)_2$ from $(1011)_2$.

Solution:

$$\begin{array}{r} \scriptstyle 0\ 2\quad\quad \\ (\not{1}\ 0\ 1\ 1)_2 \\ -(\ \ 1\ 1\ 1)_2 \\ \hline (1\ 0\ 0)_2 \end{array}$$

Example 21:

Evaluate $(110001)_2 - (11101)_2$

Solution:

$$\begin{array}{r} \scriptstyle 2\ 1\quad\quad\quad \\ \scriptstyle 0\ 0\ 2\ 2\quad\quad \\ (\not{1}\ \not{1}\ \not{0}\ 0\ 0\ 1)_2 \\ -(\ \ 1\ 1\ 1\ 0\ 1)_2 \\ \hline (1\ 0\ 1\ 0\ 0)_2 \end{array}$$

3.2.2(c) Multiplication of Numbers in Binary System

The numbers in binary system are multiplied in the following way.

$0 \times 0 = 0,\ 0 \times 1 = 0,\ 1 \times 0 = 0,\ 1 \times 1 = 1$

Note that multiplication in binary system is similar as in decimal system, we explain multiplication of binary numbers by the following table.

×	0	1
0	0	0
1	0	1

Example 22:

Multiply the following binary numbers.

(i) $(101)_2, (11)_2$ **(ii)** $(1101)_2, (101)_2$

Solution:

(i)

$$\begin{array}{r} (1\ 0\ 1)_2 \\ \times\ \ (1\ 1)_2 \\ \hline (1\ 0\ 1)_2 \\ (1\ 0\ 1\ 0)_2 \\ \hline (1\ 1\ 1\ 1)_2 \end{array}$$

(ii)

$$\begin{array}{r} (1\ 1\ 0\ 1)_2 \\ \times\ \ (1\ 0\ 1)_2 \\ \hline (1\ 1\ 0\ 1)_2 \\ (0\ 0\ 0\ 0\ 0)_2 \\ (1\ 1\ 0\ 1\ 0\ 0)_2 \\ \hline (1\ 0\ 0\ 0\ 0\ 0\ 1)_2 \end{array}$$

Example 23: Evaluate

$$(1\ 0\ 1\ 1\ 0\ 1\ 1\ 0)_2 \times (1\ 1\ 0\ 1\ 1)_2$$

Solution:

$$\begin{array}{r} (1\ 0\ 1\ 1\ 0\ 1\ 1\ 0)_2 \\ \times\ \ (1\ 1\ 0\ 1\ 1)_2 \\ \hline (1\ 0\ 1\ 1\ 0\ 1\ 1\ 0)_2 \\ (1\ 0\ 1\ 1\ 0\ 1\ 1\ 0\ 0)_2 \\ (0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0)_2 \\ (1\ 0\ 1\ 1\ 0\ 1\ 1\ 0\ 0\ 0\ 0)_2 \\ (1\ 0\ 1\ 1\ 0\ 1\ 1\ 0\ 0\ 0\ 0\ 0)_2 \\ \hline (1\ 0\ 0\ 1\ 1\ 0\ 0\ 1\ 1\ 0\ 0\ 1\ 0)_2 \end{array}$$

Explanation: While adding the digits when sum of digits becomes greater than 3 (as in 6^{th} column), proceed as follows.

$1 + 1 + 1 + 1 + 1 = 5$ which is 101 in binary system. Place 1 below the 6^{th} column and carry $(10)_2$ or 2 in the 7^{th} column.

Now in 7^{th} column $1 + 1 + 2 = 4$ which is 100 in binary system. Place 0 below the 7^{th} column and carry $(10)_2$ or 2 in the 8^{th} column and so on.

Exercise 3.3

1. Simplify the following.

(i) $(111)_2 + (101)_2$ **(ii)** $(1011)_2 + (1010)_2$

(iii) $(111100111)_2 + (1001101110)_2$

(iv) $(110110110110)_2 + (10110111011)_2$

(v) $(101001100010)_2 + (1111011110)_2$

2. Simplify the following.

(i) $(111)_2 - (110)_2$ (ii) $(1101)_2 - (111)_2$

(iii) $(1101000)_2 - (111011)_2$

(iv) $(101100110)_2 - (10101101)_2$

(v) $(1000000)_2 - (111111)_2$

3. Simplify the following.

(i) $(1011)_2 + (1101)_2 + (1110)_2$

(ii) $(1101)_2 - (10101)_2 + (100101)_2$

(iii) $(110101)_2 - \{(1010)_2 + (1110)_2\}$

(iv) $(100000)_2 - \{(11101)_2 - (10011)_2\}$

4. Evaluate the following binary numbers.

(i) $(111)_2 \times (11)_2$ (ii) $(10101)_2 \times (1010)_2$

(iii) $(11011)_2 \times (10110)_2$ (iv) $(1001)_2 \times (1011010)_2$

(v) $(1010101010)_2 \times (10101)_2$ (vi) $(101100101)_2 \times (11010)_2$

(vii) $(101)_2 \times (11)_2 \times (111)_2$ (viii) $\{(110)_2 \times (1101)_2\} \times (11)_2$

Reading

3.2.3 Adding, Subtracting and Multiplying Numbers in Base Five System

3.2.3(a) Addition of Numbers in Base Five System

We know that there are five digits 0, 1, 2, 3, 4 in the base five system. While adding the digits in the base five system if sum of the two or more digits is greater than 4 then the resultant number is converted into base five system. The digit at unit place is written below the column while other digit is added to the digits in the next column.

Example 24: Add the numbers $(4423)_5$, $(4344)_5$

Solution:

$$\begin{array}{r} {\scriptstyle 1\ 1\ 1\ \ } \\ (4\ 4\ 2\ 3)_5 \\ +\ (4\ 3\ 4\ 4)_5 \\ \hline (1\ 4\ 3\ 2\ 2)_5 \end{array}$$

Procedure

(i) In the first column 4 + 3 = 7 which is equal to 12 in the base five system.

(ii) Write 2 below the first column and carry 1 to the second column.

(iii) In the second column 2 + 4 + 1= 7 and repeat the process applied in the first column.

(iv) In the third column, 4 + 3 + 1= 8 which is equal to 13 in the base five system.

(v) Write 3 below the third column and carry 1 to the fourth column.

(vi) Finally add 4 + 4 + 1 = 9 which is equal to 14 in base five system.

Example 25:

Find the sum of $(12340)_5$, $(24213)_5$ and $(44334)_5$

Solution:

$$\begin{array}{r} {\scriptstyle 2\ 1\ 1\ 1\ \ \ } \\ (1\ 2\ 3\ 4\ 0)_5 \\ (2\ 4\ 2\ 1\ 3)_5 \\ +\ (4\ 4\ 3\ 3\ 4)_5 \\ \hline (1\ 4\ 1\ 4\ 4\ 2)_5 \end{array}$$

3.2.3(b) Subtraction of Numbers in Base Five System

In base five system when a digit having greater value is subtracted from the smaller digit, we borrow 1 from the digit of next column which becomes 5 right in the same way in the decimal system, we borrow 1 from the digit of next column, it becomes 10.

For example we subtract $(22)_5$ from $(41)_5$

$$\begin{array}{r} {\scriptstyle 3\ \ 5\ \ } \\ (\not{4}\ 1)_5 \\ -\ (2\ 2)_5 \\ \hline (1\ 4)_5 \end{array}$$

We cannot subtract 2 from 1 in the first column. Therefore we borrow 1 from the second column which will become 5 in the first column. Now adding 5 + 1 = 6 and subtracting 2 from 6, we have 6 – 2 = 4. In the second column after 1 is borrowed, we have 4 – 1 = 3. Finally 3 – 2 = 1.

Example 26: Simplify

(i) $(4211)_5 - (2314)_5$ **(ii)** $(3202)_5 - (1243)_5$

Solution:

(i)

$$\begin{array}{r} \ \ 5\ \ \ \ \\ 3\ 1\ 5\ 5\ \ \\ (\not{4}\ \not{2}\ \not{1}\ 1)_5 \\ -\ (2\ 3\ 1\ 4)_5 \\ \hline (1\ 3\ 4\ 2)_5 \end{array}$$

(ii)

$$\begin{array}{r} 5\ 4\ \ \ \ \\ 2\ 1\ \not{5}\ 5\ \ \\ (\not{3}\ \not{2}\ \not{0}\ 2)_5 \\ -\ (1\ 2\ 4\ 3)_5 \\ \hline (1\ 4\ 0\ 4)_5 \end{array}$$

3.2.3(c) Multiplication of Numbers in Base Five System

While multiplying the two digits in base five system, if product of two digits is greater than 4 then it is converted into an equivalent number in base five system.

e.g. $4 \times 2 = 8$ which is equal to 13 in base five system.

$\therefore \quad (4)_5 \times (2)_5 = (13)_5$

Example 27: Evaluate $(34)_5 \times (23)_5$

Solution:

$$\begin{array}{r} (3\ 4)_5 \\ \times\ (2\ 3)_5 \\ \hline (2\ 1\ 2)_5 \\ +\ (1\ 2\ 3\ 0)_5 \\ \hline (1\ 4\ 4\ 2)_5 \end{array}$$

Procedure

1. In the first line of multiplication.

(i) $4 \times 3 = 12 = (22)_5$.

Write 2 below and carry over 2.

(ii) $3 \times 3 = 9 + 2 = 11$ which is equal to $(21)_5$.

Write 21 below second column.

2. In the second line of multiplication.

(i) $4 \times 2 = 8 = (13)_5$

Write 3 below second column and carry over 1.

(ii) $3 \times 2 = 6 + 1 = 7$ which is equal to $(12)_5$. Write 12 in the third position.

3. In the third line of addition, add the two lines of multiplication.

Example 28:

Multiply $(3432)_5$ and $(20243)_5$

Solution:

$$\begin{array}{r} (2\,0\,2\,4\,3)_5 \\ \times\ (3\,4\,3\,2)_5 \\ \hline (4\,1\,0\,4\,1)_5 \\ (1\,1\,1\,3\,3\,4\,0)_5 \\ (1\,3\,2\,1\,3\,2\,0\,0)_5 \\ (1\,1\,1\,3\,3\,4\,0\,0\,0)_5 \\ \hline (1\,3\,1\,3\,1\,2\,1\,3\,1)_5 \end{array}$$

Exercise 3.4

Evaluate the following.

1. $(32)_5 + (12)_5$
2. $(34)_5 + (43)_5$
3. $(333)_5 + (222)_5$
4. $(1234)_5 + (4443)_5$
5. $(10223)_5 + (31244)_5$
6. $(432434)_5 + (243434)_5$
7. $(3024)_5 + (2432)_5 + (2203)_5$
8. $(34)_5 - (23)_5$
9. $(33)_5 - (24)_5$
10. $(342)_5 - (234)_5$
11. $(1000)_5 - (333)_5$
12. $(22222)_5 - (4444)_5$
13. $(323232)_5 - (133333)_5$
14. $(40404)_5 - (3030)_5$
15. $(123)_5 + \{(4302)_5 - (1234)_5\}$
16. $\{(2001)_5 - (1233)_5\} - (14)_5$
17. $(11111)_5 - \{(4030)_5 - (1222)_5\}$
18. $(23)_5 \times (34)_5$
19. $(302)_5 \times (24)_5$
20. $(222)_5 \times (432)_5$
21. $(3022)_5 \times (1443)_5$
22. $(40343)_5 \times (3424)_5$

Reading

3.2.4 Adding, Subtracting and Multiplying Numbers in Base Eight System

3.2.4(a) Addition of Numbers in Base Eight System

We know that there are seven digits 0,1, 2, 3, 4, 5, 6, 7 in the base eight system. While adding the digits in the base eight system if sum of the two or more digits is greater than 7 then the resultant number is converted into base eight system. The digit at unit place is written below the column while other digit is added to the digits in the next column.

Example 29: Add the following numbers.

$(4735)_8$, $(2364)_8$

$$\begin{array}{r} {\scriptstyle 1\ 1\ 1\ \ } \\ (4\ 7\ 3\ 5)_8 \\ +\ (2\ 3\ 6\ 4)_8 \\ \hline (7\ 3\ 2\ 1)_8 \end{array}$$

Solution:

Procedure

(i) In the first column 5 + 4 = 9 which is equal to 11 in the base eight system.

(ii) Write 1 below the first column and carry 1 to the second column.

(iii) In the second column 3 + 6 + 1= 10 which is equal to 12 in the base eight system. Write 2 in the second column and carry 1 to the third column.

(iv) In the third column, 7 + 3 + 1= 11 which is equal to 13 in the base eight system.

(v) Write 3 below the third column and carry 1 to the fourth column.

(vi) Finally add 4 + 2 + 1 = 7

Example 30:

Find the sum of $(16570)_8$, $(27613)_8$ and $(45223)_8$

Solution:

$$\begin{array}{r} {\scriptstyle 2\ 1\ 1\ \ \ \ \ \ } \\ (1\ 6\ 5\ 7\ 0)_8 \\ (2\ 7\ 6\ 1\ 3)_8 \\ +\ (4\ 5\ 2\ 2\ 3)_8 \\ \hline (1\ 1\ 3\ 6\ 2\ 6)_8 \end{array}$$

3.2.4(b) Subtraction of Numbers in Base Eight System

In base eight system when a digit having greater value is subtracted from the smaller digit, we borrow 1 from the digit of next column which becomes 8 right in the given column.

For example we subtract $(46)_8$ from $(61)_8$.

$$\begin{array}{r} {\scriptstyle 5\ \ 8\ \ } \\ (\not{6}\ 1)_8 \\ -\ (4\ 6)_8 \\ \hline (1\ 3)_8 \end{array}$$

We cannot subtract 6 from 1 in the first column. Therefore we borrow 1 from the second column which will become 8 in the first column. Now adding 8 + 1 = 9 and subtracting 6 from 9, we have 9 – 6 = 3. In the second column after 1 is borrowed, we have 6 – 1 = 5. Finally 5 – 4 =1.

Example 31: Simplify

(i) $(7203)_8 - (5514)_8$ **(ii)** $(52641)_8 - (12732)_8$

Solution:

(i)

$$\begin{array}{r} 8\;\;7\;\;\;\; \\ 6\;\;1\;\;8\;\;8\;\; \\ (\not{7}\;\not{2}\;\not{0}\;3)_8 \\ -\;(5\;5\;1\;4)_8 \\ \hline (1\;4\;6\;7)_8 \end{array}$$

(ii)

$$\begin{array}{r} 8\;\;\;\;\;\;\;\;\;\; \\ 4\;\;1\;\;8\;3\;\;8\;\; \\ (\not{5}\;\not{2}\;6\;\not{4}\;1)_8 \\ -\;(1\;2\;7\;3\;2)_8 \\ \hline (3\;7\;7\;0\;7)_8 \end{array}$$

3.2.4(c) Multiplication of Numbers in Base Eight System

While multiplying the two digits in octal number system, if product of two digits is greater than 7 then it is converted into an equivalent number in base eight system.

e.g. $4 \times 5 = 20$

Which is equal to 24 in base eight system.

$\therefore \quad (4)_8 \times (5)_8 = (24)_8$

Example 32:

Evaluate $(64)_8 \times (25)_8$

Solution:

$$\begin{array}{r} (6\;4)_8 \\ \times\;(2\;5)_8 \\ \hline (4\;0\;4)_8 \\ +\;(1\;5\;0\;0)_8 \\ \hline (2\;1\;0\;4)_8 \end{array}$$

Procedure

1. In the first line of multiplication.

(i) $5 \times 4 = 20 = (24)_8$.

Write 4 below and carry over 2.

(ii) $5 \times 6 = 30 + 2 = 32$ which is equal to $(40)_8$.

Write 40 below second column.

2. In the second line of multiplication.

   (i) $2 \times 4 = 8 = (10)_8$

   Write 0 below second column and carry over 1.

   (ii) $2 \times 6 = 12 + 1 = 13$ which is equal to $(15)_8$.

   Write 15 in the third position.

3. Finally add the two lines of multiplication.

Example 33:

Multiply $(2642)_8$ and $(53047)_8$.

Solution:

$$
\begin{array}{r}
(5\,3\,0\,4\,7)_8 \\
\times\ (2\,6\,4\,2)_8 \\
\hline
(1\,2\,6\,1\,1\,6)_8 \\
(2\,5\,4\,2\,3\,4\,0)_8 \\
(4\,0\,2\,3\,5\,2\,0\,0)_8 \\
(1\,2\,6\,1\,1\,6\,0\,0\,0)_8 \\
\hline
(1\,7\,1\,2\,4\,3\,6\,5\,6)_8
\end{array}
$$

Exercise 3.5

Evaluate the following.

1. $(35)_8 + (42)_8$
2. $(76)_8 + (34)_8$
3. $(555)_8 + (444)_8$
4. $(1524)_8 + (4662)_8$
5. $(10223)_8 + (31244)_8$
6. $(765432)_8 + (234567)_8$
7. $(5074)_8 + (2642)_8 + (1153)_8$
8. $(75)_8 - (66)_8$
9. $(55)_8 - (26)_8$
10. $(475)_8 - (277)_8$
11. $(1000)_8 - (444)_8$
12. $(33333)_8 - (5555)_8$
13. $(545454)_8 - (244422)_8$
14. $(60606)_8 - (4040)_8$
15. $(153)_8 + \{(6304)_8 - (2534)_8\}$
16. $\{(7007)_8 - (4244)_8\} - (30)_8$
17. $(11111)_8 - \{(2070)_8 + (1666)_8\}$
18. $(43)_8 \times (56)_8$
19. $(307)_8 \times (63)_8$
20. $(555)_8 \times (314)_8$
21. $(3077)_8 \times (1446)_8$
22. $(40573)_8 \times (5403)_8$
23. $(127)_8 \times (21)_8 \times (44)_8$

Reading

3.2.5 Adding, Subtracting and Multiplying the Numbers with Different Bases

To solve the problems containing numbers with different bases, it is better to convert all numbers into one of the number systems discussed before. Since we are more familiar with decimal system. Therefore it is more convenient to convert all the numbers in the given problem into decimal system. Finally the resultant number can be converted into required number system.

Example 34:

Evaluate $(2301)_5 + (110110)_2 - (165)_8$

Solution:

$$(2301)_5 = (2 \times 5^3) + (3 \times 5^2) + (0 \times 5^1) + (1 \times 5^{o})$$
$$= (2 \times 125) + (3 \times 25) + (0 \times 5) + (1 \times 1)$$
$$= 250 + 75 + 0 + 1 \quad = \quad 326$$

$$(110110)_2 = (1 \times 2^5) + (1 \times 2^4) + (0 \times 2^3) + (1 \times 2^2) + (1 \times 2^1) + (0 \times 2^{o})$$
$$= (1 \times 32) + (1 \times 16) + (0 \times 8) + (1 \times 4) + (1 \times 2) + (0 \times 1)$$
$$= 32 + 16 + 0 + 4 + 2 + 0 \quad = \quad 54$$

$$(165)_8 = (1 \times 8^2) + (6 \times 8^1) + (5 \times 8^{o})$$
$$= (1 \times 164) + (6 \times 8) + (5 \times 1)$$
$$= 164 + 48 + 5$$
$$= 217$$

Now $(2301)_5 + (110110)_2 - (165)_8$

$$= 326 + 54 - 217$$
$$= 163$$

Example 35:

Evaluate and express the resultant number into base five system.

$3080 - (100111)_2 \times (222)_5$

Solution:

$(100111)_2 = (1\times 2^5) + (0\times 2^4) + (0\times 2^3) + (1\times 2^2) + (1\times 2^1) + (1\times 2^o)$

$= (1\times32) + (0\times16) + (0\times 8) + (1\times 4) + (1\times 2) + (1\times 2) + (1\times1)$

$= 32 + 0 + 0 + 4 + 2 + 1$

$= 39$

$(222)_5 = (2\times 5^2) + (2\times 5^1) + (2\times 5^o)$

$= (2\times 25) + (2\times 5) + (2\times 1)$

$= 50 + 10 + 2$

$= 62$

$\therefore \quad 3080 - (100111)_2 \times (222)_5$

$= 3080 - 39\times 62$

$= 2418$

$= 662$

5	672
5	134 – 2
5	26 – 4
5	5 – 1
	1 – 0

Now we convert 672 into base five system.

Hence $672 = (10142)_5$

Exercise 3.6

Evaluate questions from 1 to 5.

1. $(101010)_2 + (2340)_5 + (67)_8$

2. $(2321)_5 - (1100110)_2 + (55)_8$

3. $(650)_8 \times (333)_5 \times (1001)_2$

4. $809 - \{(111001)_2 - (3240)_5 + (1041)_8\}$

5. $90 + \{(1110)_2 \times (234)_5 - (472)_8\}$

6. Convert the following numbers into base five and eight systems.

(i) $(110011)_2$ **(ii)** $(100110110)_2$

7. Convert the following numbers into binary as well as octal number system.

(i) $(324)_5$ **(ii)** $(4030)_5$

8. Convert the following numbers into binary as well as base five system.

(i) $(734)_8$ **(ii)** $(1052)_8$

Review Exercise 3

Encircle the best answer in the following.

1. (i). Base ten system of numbers is also called _________ system.

(a) decimal (b) octal (c) binary (d) denary

(ii). Digits used in base 2 are:

(a) 0, 2 (b) 0, 1, 2 (c) 0, 1 (d) 1, 2

(iii). $2 \times 10^2 + 3 \times 10^1 + 5 \times 10^o =$

(a) 285 (b) 235 (c) 532 (d) 253

(iv). $(4)_5 + (3)_5 =$

(a) $(7)_5$ (b) $(10)_5$ (c) $(11)_5$ (d) $(12)_5$

(v). $(12)_8 - (4)_8 =$

(a) $(6)_8$ (b) $(5)_8$ (c) $(7)_8$ (d) $(8)_8$

(vi). $(101)_2 \times (11)_2 =$

(a) $(1001)_2$ (b) $(111)_2$ (c) $(1111)_2$ (d) $(1000)_2$

(vii). $(110)_2 = (\_\_\_\_\_\_\_\_\_\_)_5$

(a) 11 (b) 10 (c) 6 (d) 12

(viii). $(247)_8 = (\_\_\_\_\_\_\_\_\_\_)_5$

(a) 1231 (b) 1321 (c) 1123 (d) 1132

(ix). $(222)_5 = (\_\_\_\_\_\_\_\_\_\_)_2$

(a) 11011 (b) 111110 (c) 111101 (d) 11110

(x). 10 + 2 in base five is

(a) 22 (b) 12 (c) 20 (d) 40

(xi). $(34)_5$ in base ten is

(a) $3 \times 5^2 + 4 \times 5^1$ (b) 19 (c) 64 (d) 20

(xii). $(3)_8 \times (3)_8 =$

(a) $(11)_5$ (b) $(12)_8$ (c) $(11)_2$ (d) $(11)_8$

(xiii). $3 \times 5^3 + 0 \times 5^2 + 1 \times 5^1 + 3 \times 5^o =$

(a) 3013 (b) 383 (c) 833 (d) 338

2. Evaluate the following.

(i). $(1001)_2 + (1111)_2 - (111)_2$

(ii). $(1011)_2 \times (111)_2 + (1111)_2$

(iii). $(4321)_5 - (1234)_5 + (3331)_5$

(iv). $(4504)_8 - (3652)_8 + (1254)_8$

3. Evaluate $(12340)_5 + (1100111)_2 - 1000$

4. Evaluate $(2401)_5 \times \{(235)_8 - (1100010)_2\}$

Key Point

- The numbers are combination of digits. In a given number, every digit has certain place value.
- We use ten digits in decimal system which are 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. In decimal system, all the place values of digits are in terms of powers of 10.
- The number system with base 2 is also known as binary system. The digits used in this system are 0 and 1. In binary system we count in bundles of two. Place values of digits in the binary system are in terms of powers of 2.
- The importance of binary system is due to the fact that it is used in digital computers, calculators, digital watches and digital balances etc.
- In the base five system we count in the bundles of 5. The digits used in base five system are 0, 1, 2, 3, 4. Place values of digits in base five system are in terms of powers of 5.
- In base eight system we count in bundles of eight. The digits used in this system are 0, 1, 2, 3, 4, 5, 6 and 7. In base eight system we count in bundles of eights.
- Place values of digits in base eight system are in terms of powers of 8. This system is also known as octal number system.

4

FINANCIAL MATHEMATICS

This is 20 days unit
(periods including homework)

After studying this unit, you will be able to:

- define compound proportion and solve real life problems involving compound proportion, partnership and inheritance.
- know about commercial banking and types of bank accounts.
- describe negotiable instruments like cheque, demand drafts and pay order.
- explain on line banking, transaction through ATM, debt and credit card.
- know about exchange of currencies.
- know about profit/markup, principal amount.
- know about types of finance and solve problems related to banking and finance.
- solve real life problems involving successive transactions related to profit, loss and discount.
- define insurance and know about life and vehicle insurance.
- solve real life problems involving insurance.
- explain income tax and solve real life problems related to it.

Reading

4.1 COMPOUND PROPORTION

4.1.1 Review

We know that a proportion is a mathematical statement showing that two ratios are equal. If a, b, c and d are in proportion then we write

a : b = c : d or a : b : : c : d

There are two types of proportion.

(i) Direct Proportion

If a, b, c, d are in direct proportion then

a : b = c : d or $\frac{a}{b} = \frac{c}{d}$

(ii) Inverse Proportion

If a, b, c, d are in inverse proportion then a : b = d : c $\Rightarrow \frac{a}{b} = \frac{d}{c}$

4.1.2 Real Life Problems involving Compound Proportion, Partnership and Inherences

(a) Compound Proportion

The combination of two or more proportions (either direct or inverse or both) is called compound proportion.

We explain the method of compound proportion with the help of examples taken from daily life.

Example 1:

15 scouts can eat 30kg fruit in 5 days. In how many days can 20 scouts eat 40kg fruit?

Solution:

Let number of days = x

Days	Fruit (kg)	Scouts
5 ↑ x	30 ↑ 40	15 ↓ 20

As quantity of fruit increases, it will be sufficient for more days. } Direct proportion

As number of scouts increase, fruit will be sufficient for less days. } Inverse proportion

$$\frac{x}{5} = \frac{40}{30} \times \frac{15}{20}$$

$$x = \frac{40}{30} \times \frac{15}{20} \times 5$$

$$x = 5$$

$\therefore$ Number of days = 5

Example 2:

A contractor got a contract to build a school building in 4 months and employed 15 workers. After 80 days only one fourth work was completed. How many more workers are required to complete the building in specified time?

Solution:

Total days = 4×30 = 120

Number of days in which work was in progress = 80

Remaining days = 120 – 80 = 40

Let total work = 4 parts

Work completed = 1 part

Remaining work = 3 parts

Let number of workers required = x

Workers	Days	Work
15 ↑	80 ↓	1 ↑
x	40	3

Less days, more workers are required } Inverse proportion

More work, more workers are required. } Direct proportion

$$\frac{x}{15} = \frac{80}{40} \times \frac{3}{1}$$

$$x = \frac{80}{40} \times \frac{3}{1} \times 15$$

$$x = 90$$

$\therefore$ More workers required = 90 – 15

= 75

(b) Partnership

A partnership is a business run by at least two persons, companies or parties in which amount of investment and time span may be different. At the end profit obtained or loss suffered is shared according to the amount and time of investment.

Example 3:

Asif, Sharjeel and Saqib started a business with an investment of Rs.70000, Rs.60000 and Rs.80000 respectively. After 6 months Saqib took his money back.

They suffered a loss of Rs.52500 at the end of the year. Find how much each person suffers.

Solution:

Asif's share = Rs.70000
Sharjeel's share = Rs.60000
Saqib's share = Rs.90000

After 6 months Saqib took his money back, so his duration of investment is (12 – 6) months = 6 months

Asif's investment	:	Sharjeel's investment	:	Saqib's investment
12×70000	:	12×60000	:	6×90000
14	:	12	:	9

Sum of elements of ratio = 14 +12 + 9 = 35

Asif's share = Rs. $\frac{14}{35} \times 52500$
= Rs.21000

Sharjeel's share = Rs. $\frac{12}{35} \times 52500$
= Rs.18000

Saqib's share = Rs. $\frac{9}{35} \times 52500$
= Rs.13500

Pay Heed
If duration of investments of two or more partners is same, then we can find the ratio between the investments without multiplying the investments by their duration of investment.

Example 4:

Alia and Lubna started a business with investments Rs.10000 and Rs.15000 respectively. After three months Fatima also joined them with the investment of Rs.20000. At the end of one year, they got a profit of Rs.48000. Find share of each.

Solution:

Alia's capital	:	Lubna's Capital	:	Fatima's capital
12×10000	:	12×15000	:	9×20000
120000	:	180000	:	180000
12	:	18	:	18
2	:	3	:	3

Sum of elements of ratio = 2 + 3 + 3

	=	8
Total amount of profit	=	Rs.48000
Alis's share	=	$\frac{2}{8} \times$ Rs.48000
	=	Rs.12000
Lubna's share	=	$\frac{3}{8} \times$ Rs.48000
	=	Rs.18000
Fatima's share	=	$\frac{3}{8} \times$ Rs.48000 = Rs.18000

(c) Inheritance

Inheritance is an important branch of the family laws of the Muslims. After the death of a person, most of his rights are transferred to his legal heirs. The property left after the payment of funeral expenses, the discharge of his debts and testament, is to be distributed according to the law of inheritance.

We shall discuss here only some important shares of heirs.

- Father, mother will get one sixth of total when deceased leaves children.
- Widow will get one eighth of total when deceased leaves children.
- If deceased is woman and has children then her husband will get one fourth of the property.
- The remaining property will be distributed among sons and daughters in the ratio 2 : 1.

While distributing the property among the inheritors, keep in mind the following points.

1. Distribute the property left over, after the payment of funeral expenses, debts and testament.
2. Amount of testament should not be more than one third of the property.

Example 5:

A person died leaving the property worth Rs.366000. Divide this property among his mother, widow, two sons and three daughters after paying a debt of Rs.14000 and funeral expenses of Rs.16000.

Solution:

Property of person died	=	Rs. 366000
Payment of debt	=	Rs. 14000
Funeral expenses	=	Rs. 16000

Remaining property = Rs. (366000 – 14000 – 16000)
= Rs. 336000

Share of mother = $\frac{1}{6} \times$ Rs.336000
= Rs. 56000

Share of widow = $\frac{1}{8} \times$ Rs.336000
= Rs. 42000

Remaining property = Rs. (336000 – 56000 – 42000)
= Rs. 238000

Ratio among shares of sons and daughters.

Son : Son : daughter : daughter : daughter

2 : 2 : 1 : 1 : 1

Sum of ratios = 2 + 2 + 1 + 1 + 1 = 7

Share of each son = $\frac{2}{7} \times$ Rs.238000
= Rs.68000

Share of each daughter = $\frac{1}{7} \times$ Rs.238000 = Rs.68000

Example 6:

A woman died leaving property worth Rs.148000. Divide her property among two sons and husband. She wished in her life to give 10% of her property to the mosque.

Solution:

Property of woman = Rs.148000

Amount given to the mosque = $\frac{10}{100} \times$ Rs.148000
= Rs.148000

Remaining property = Rs.148000 – 14800
= Rs.133200

Share of husband = $\frac{1}{4} \times$ Rs.133200
= Rs.33300

Remaining property = Rs.133200 – Rs.33300

$= \text{Rs.}99900$

Each son will get $= \frac{1}{2} \times \text{Rs.}99900 = \text{Rs.}49950$

Exercise 4.1

1. A truck driver charges Rs.2000 for 10 km with a load of 800kg. What will he charge if truck covers a distance of 8km with a load of 600kg?
2. 10 men can repair 5 machines in 2 days. How many machines are repaired by 8 men in 3 days?
3. 12 men can do a work in 8 days. How many men can do three times of that work in 16 days?
4. 100 hens eat one fourth quantity of feed in 21 days. In how many days 84 hens will eat remaining feed?
5. In a factory 49 workers made 1050 articles in 10 hours. One day 4 workers were absent. In how many hours can remaining workers make 1350 articles?
6. For 28 scouts 168kg food is sufficient for 6 days. For how many days 144kg food is sufficient if 8 more scouts join them.
7. Hassan, Akbar and Asghar started a business by investing Rs.10000, Rs.12000 and Rs.15000 respectively. Asghar took his money back after 10 months. Find the share of each in the profit of Rs.27600 after one year.
8. Qasim, Kashif and Tahir shared Rs.1200, Rs.1500 and Rs.1800 respectively for a business. After 5 months Kashif took his money back. At the end of 10 months they suffered a loss of Rs.1875. Find the share of each in loss.
9. X started the business with the investment Rs.3500. After three months Y joined him with the investment of Rs.3000. Then after two months Z also joined them with the investment of Rs.4000. They closed their business after 5 months. Find share of each in the profit of Rs.15200.
10. Divide a property worth Rs.300000 among two sons, a daughter and a widow after paying the funeral expenditures of Rs.10000.
11. A man died leaving a house worth Rs.560000 and a shop worth Rs.260000. Divide his property among his parents, widow and a daughter after paying a debit of Rs.20000.

12. Mr. Akbar died last Sunday. He had the following property:
    (i) 10 Kanals land @ Rs.30000 per Kanal.
    (ii) house worth Rs.800000.

    Divide this property among his widow and a son.
13. A person died leaving a property worth Rs.500000. Divide his property among his widow, a daughter and two sons.
14. Ms Sajida is ill now-a-days. She wants to distribute her property worth Rs.850000 among her husband, two sons and two daughters in her life. Can you help her? (Share of husband is one fourth of property.)

Reading

4.2 COMMERCIAL BANKING

4.2.1(a) Commercial Bank Deposits/ Accounts

The commercial bank deposits are of following types.

i. Current deposit accounts
ii. PLS saving accounts
iii. PLS term deposit accounts
iv. Foreign currency accounts

i. Current Deposit Accounts

In current accounts, the customers can deposit their surplus money in the banks on the basis of no profit no loss. The bank does not deduct Zakat or wealth tax on the money deposited in current accounts.

ii. PLS Saving Accounts

The customers can also open their accounts on profit and loss sharing basis. The profit rate is not fixed for the money deposited in PLS saving accounts. The banks are authorized to deduct Zakat on the amount deposited in PLS saving accounts.

iii. PLS Term Deposit Accounts

In PLS term deposit accounts, the customers can deposit their money for a fixed period like six months, one year, two years etc. The amount deposited in PLS term deposit account cannot be withdrawn before the expiry of the fixed period or without a due notice. In PLS term deposit accounts the profit rate is announced after the completion of fixed period.

iv. Foreign Currency Accounts

In Pakistan, the banks offer foreign currency accounts in dollars, pounds, mark, yen and euro etc. The banks did not charge any deduction like Zakat or wealth tax etc. on the amount deposited in foreign currency accounts.

4.2.1(b) Negotiable Instruments

The negotiable instruments are of following types:

i. Cheque ii. Demand Draft

iii. Pay Order

> A document which enables the person holding it, to get a specified sum of money from the bank is called negotiable instrument.

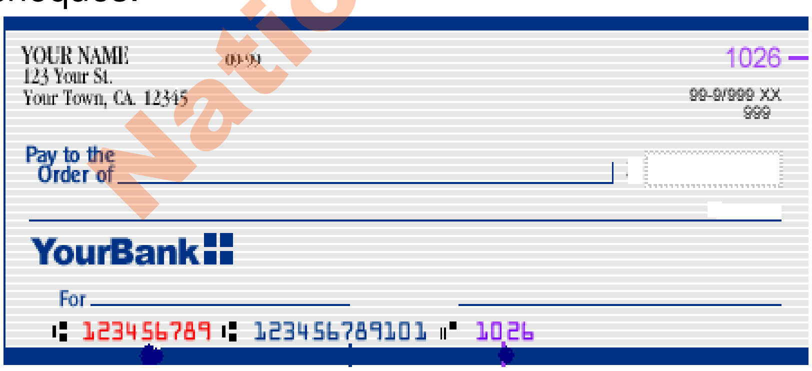
i. Cheque

The bank provides its customer a cheque book consisting of 25, 50 or more leaves. Main purpose of cheque is to withdraw money from the bank. Each leaf of cheque book consists of two parts, main and counterpart.

- The main part of cheque is the actual cheque which is submitted in the bank to withdraw the money.
- The counter part of the cheque is just for customer's record.

ii. Demand Draft

Demand drafts are used by individuals to make transfer payments from one bank account to another. Demand draft is relatively secure method for cashing cheques.



- The demand drafts do not require a signature in order to be cashed like cheque.
- Demand draft is used to pay the money outside the city.

iii. Pay Order

Pay Order is another payment instrument which is used by the banks for payments on behalf of their customers. Pay order is guaranteed by the bank for its full value and is similar to a demand draft but is used for local payments.

4.2.2 Online Banking

It is used to perform transactions over the internet through a bank or credit union etc. Customers can use this facility of banking outside the bank hours and from anywhere where internet access is available. Online banking usually offers the following features:

(i) Funds transfer (ii) Bank statements (iii) Utility bill payments
(iv) Investments (v) Loan and repayments

Auto Teller Machine (ATM)

It is a computerized telecommunication device. It facilitates customers of a bank to make transactions any time.

- Using ATM, customers can access their bank accounts for cash withdrawals and to check their account balances.
- Through many ATMs one can also deposit cash or cheque, transfer money, pay bills, purchase goods and services.

ATM Card

To make transactions through ATMs, a card is required known as ATM card. It is made of plastic with a magnetic strip that contains a unique card number and some information related to security and expiry date. Security is provided by the customer entering a personal identification number commonly known as PIN.

Credit Card

A credit card is a small plastic card with magnetic strip. It is issued by a bank to its customer giving him an option to borrow funds, usually at point of sale. Borrowing limits of credit card are pre-set according to the individual's.

Debit Card

A debit card is also a small plastic card with magnetic strip. It is issued by a bank which allows bank clients access to their account to pay for goods and services.

- An ATM card can also be used as debt card.

Example 7:

Anwar Khan has Rs.90000 in his account. He withdraws an amount of Rs.20000 by using demand draft. The bank deducts service charges @ 1.5 % on the deduction. Next day he deposits Rs.10000 in his account. Find the service charges and balance in the account.

Solution:

Total amount = Rs.90000

Amount withdrawn = Rs.20000

Rate of service charges = 1.5%

Service charges = Rs. $20000 \times \frac{1.5}{100}$

= Rs.300

Balance in the account = Rs.(90000 – 20000 – 300 + 10000)

= Rs.79700

Exercise 4.2

1. A man opens PLS saving account in a bank by depositing Rs.100000. The bank announces 6.5 % profit for first six months and 7 % for the next six months. What is the balance in the account after one year?

   (**Hint** : Add the profit of 1st 6 months in Rs.100000, then calculate profit for next 6 months.)

2. Mr. Faroz Khan deposited Rs.500000 in PLS term deposit account for two years. If rate of profit announced by the bank is 10% yearly,find the amount received by him after deduction of Zakat?

3. Akram has Rs.65000 in his account. He withdraws an amount of Rs.16500 by using demand draft. The bank deducts service charges @ 2 % on the deduction. Find the service charges and balance in the account.

4. A school sent registration of 150 students for grade IX in the board amounting Rs.45000 with an additional amount of 1.7 % for pay order. Find the total amount paid by the school and by each student. Write answer in whole figures.

5. Aleena has Rs.80000 in her account. She withdraws 20 % of her balance through ATM card from the same branch and Rs.5000 from another bank's ATM. She also pays utility bills of Rs.1700 through the card. Find the remaining amount in the account.

(**Hint** : Other bank deducts Rs.20 for each transaction.)

Reading

4.2.3 Conversion of Currencies

We exchange the currencies when we want

- to do a business with foreign countries' people.
- to live in other country
- to go for Ummarah or Hajj in Saudia.

Some important countries and their currencies

Country	Currency	Symbol
Saudi Arabia	Rial	SAR
Pakistan	Rupees	Rs
America (U.S.A)	Dollar	$
England (U.K)	Pound	£
China	Yuan	元
German	Euro	€
France	Euro	€
Japan	Yen	¥

Relationships between Different Currencies in 2014

> **Note:**
> These rates for currency exchange are not fixed. They change daily according the economic development of the country. So, we have to check their daily currency rates through computer (internet) or through newspapers.

1 Saudi Riyal	= 28 Rupees
1 Dollar (USA)	= 104 Rupees
1 Pound (UK)	= 160 Rupees
1 Euro	= 135 Rupees
1 Yen	= 1.5 Rupees
1 Dollar	= 0.75 Euro
1 Pound	= 2.1 Dollar
1 Dollar	= 123.7 Yen

Example 8:

Hameed wants to go for Ummarah. Before going to Makkah, he wants to exchange his money into Saudia's currency. If he has Rs.200000, then how much Riyal will he get in exchange of his money. (1 Riyal = 28 Rupees)

Solution:

28 Rupees = 1 Riyal

1 Rupee = $1 \div 28$ Riyals

= 0.0357 Riyals

200000 Rupees = 0.0357×200000 Riyals

= 7140 Riyals

Example 9:

Amir is working in America as a salesman. He came to Pakistan for two months last Sunday. If he brought 4500 Dollars with him, then how much will it be in Rupees. (1 dollar = 105 Rupees)

Solution:

1 Dollar = 105 Rupees

So 4500 Dollar = 105×4500 Rupees

= 472500 Rupees

Example 10:

American government sold his new five F–16 bombard aircrafts to France. For $905000. How much Euro will France pay to America.($ 1= 0.73 Euro)

Solution:

1 Dollar = 0.73 Euro

905000 Dollars = 0.73×905000

= 660650 Euro

Exercise 4.3

1. Convert the following foreign currencies into Pakistani Rupees.
   (a) $ 26 (b) 352 Riyal (c) 199 Euro
2. Convert the following to Japanese Yen.
   (a) $ 88 (b) 500 Euro (c) Rs.666
3. Convert the following into U.S. Dollars
   (a) Rs. 2000000 (b) 3000 Euro (c) 4200 Riyal
4. The exchange rate between the Pound and Yen during a particular day was in the ratio 1: 123.63.
   (a) How many Yens would be equivalent to 610 Pounds?
   (b) How many Pounds would be equivalent to 9360 Yens?
5. A man bought different items from a super market in Saudi Arabia and he got a bill from the accountant which was in Riyals. Convert price of each item into Rupees and then tell the total amount in Pakistani Rupees also.

Items	Prices
1. Rice (10kg)	35 Riyals
2. Sugar (5kg)	26 Riyals
3. Oil tin (5kg)	30 Riyals
4. Shampoo bottles (2.5 ℓ)	10 Riyals
5. Biscuit pack (2 pack)	15 Riyals
6. Jam bottle	12 Riyals
Total	**106.5 Riyals**

Reading

4.2.4 Profit/Markup

Principal amount

The amount deposited in the bank for certain period of time is called principal amount.

Profit

When we deposit money in a bank for a certain period of time, the bank gives us some extra amount, along with the original amount. This extra amount is called profit.

Amount

The money received from the bank after certain period of time is called amount.

Amount = Principal amount + Profit

Markup

When we borrow money from the bank, we have to pay some extra money after a certain period of time, this extra money is called Markup.

The amount given to bank is calculated as

Amount = Principal amount + Markup

Formula for Profit/Markup

If the principal amount (P) is deposited or borrowed at the rate (R%) per annum for T years then the profit/markup can be calculated by the following formula:

$$Profit/Markup = \frac{PRT}{100}$$

Example 11:

Asad borrowed Rs.300000 at the rate 10% per annum and agreed to give markup Rs.90000. After how many years will he be able to repay all money?

Solution:

P = Rs.300000 R = 10 %

Markup = 90000 T = ?

Using the formula; Markup $= \frac{PRT}{100}$

$90000 = \frac{300000 \times 10 \times T}{100}$

$\frac{90000 \times 100}{300000 \times 10} = T$

T = 3

Therefore time taken = 3 years

Example 12:

There is profit of Rs.2100 at the rate of 8 % for 7 years on some principal amount. Find principal amount.

Solution:

Profit = Rs.2100, R = 8 %, T = 7, P = ?

As Profit $= \frac{PRT}{100}$

$\therefore$ 2100 $= \frac{P \times 8 \times 7}{100}$

$P = \frac{2100 \times 100}{8 \times 7} = 3750$

$\therefore$ Principal = Rs.3750

Amount = Rs.3750 + Rs.2100

= Rs. 5850

Exercise 4.4

1. Urooj deposited Rs.55000 in a bank and received a cash of Rs.64900 after 3 years. Find the rate of profit per annum.
2. Abdullah borrowed Rs.32000 at the rate of 5% for 3 years. Find markup and amount paid by him.
3. Mrs. Aftab got an amount worth Rs.50000 with a markup of Rs.10000 for 5 years. Find the rate of markup.
4. Yousha got a profit of Rs.3000 @ 10% for 3 years from a bank. Find the amount deposited by him.
5. Urooba deposits Rs.90000 in a bank @ 4.5 %. After how many years will she get an amount of Rs.118350?
6. Fatima invested Rs.60000 @ 9 % for 3 years and Rs.40000 @ 8 % for 4 years. Find total profit got by her.
7. Mr. Shafi Ullah bought a car through a bank. The original price of car is Rs.450000. While he paid Rs.544500 in 3 years. Find markup and rate of markup. Also find his monthly installment.

Reading

4.2.5 Types of Finance

There are many types of finances which banks and other financing companies provide to their clients. Few of them are given below.

i. Overdraft (OD)
ii. Running Finance (RF)
iii. Demand Finance (DF)
iv. Leasing

(i) Overdraft (OD)

It is a special type of loan provided by the banks to their account holders. The account holders can draw a limited amount from their accounts even they have zero balance. In this situation the account is said to be overdrawn. It is repayable at any time as per demand by the bank. If you have an overdraft account, your bank will receive checks which would otherwise bounce. As with any loan, you pay markup on the outstanding balance of an overdraft loan. The banks also charge a daily fee on overdrawn accounts.

(ii) Running Finance (RF)

This is a short term facility which is granted to the borrower to enable him meeting his day to day funding needs. An agreed limit is sanctioned by the bank and the borrower is allowed to draw that amount through his current account.

Once the loan limit is approved, the borrower is free to withdraw amounts to the extent of that limit. The borrower can withdraw and repay the amount as many times as he wishes to but he has to pay markup on the amount which he has actually used.

(iii) Demand Finance (DF)

It is a type of loan given by the banks to their customers which is payable on the demand of bank any time. Demand finance may either be short term or long term; however, its repayment is done normally through installments. The bank can demand to return the loan any time.

Example 13:

Musa's account became overdrawn. He drew Rs.3000 through cheque. The bank deducted Rs.5 daily as overdraft fee. The markup rate is 5% monthly. Find the amount payable after one month.

Solution:

Loan = Rs.3000

Number of days = 30

Overdraft fee = Rs. 5 × 30 = Rs.150

Markup rate = 5 % per month

Amount of markup = Rs. 3000 × $\frac{5}{100}$ = Rs.150

Total amount payable = Rs.(3000 + 150 + 150) = Rs.3300

Example 14:

Mr. Afzal got Rs.20000 from his bank through running finance facility to pay the utility bills. He agreed to pay a markup @ 9 % per month. Find the amount payable after six months.

Solution:

Loan = Rs.20000

Number of months = 6

Markup rate = 9 % per month

Amount of markup = Rs.20000 × $\frac{9}{100}$ × 6

= Rs.10800

Total amount payable = Rs.(20000 + 10800)

= Rs.30800

Food for Thought
The lessee is the receiver of the services or the assets under the lease contract and the lesser is the owner of the assets.

(iv) Lease

It is an agreement between two parties giving one party the right to use the asset (motor vehicles, houses etc) for a period of time.
In this agreement, the lessee (user) pays the lesser (owner) for use of an asset.

Leasing/Financing of Motor Vehicle

Vehicle leasing is the leasing of a motor vehicle for a fixed period of time. It is commonly offered by dealers, banks or other finance companies as an alternative to vehicle purchase and is widely used by business men. In our country, common people are also availing this facility.

Down Payment

The amount payable by the customer to the owner at the time of agreement is called down payment.

Example 15:

Hamna wants to lease a car through a bank for a period of 3 years. The price of car is Rs.550000. She agrees to pay an amount of Rs.145000 at the time of agreement. If the interest rate is 15%, then calculate monthly installment and total amount paid by her to the bank.

Solution:

Price of car = Rs.550000

Down payment = Rs.145000

P = loan = Rs.(550000 – 145000)

= Rs.405000

R = interest rate = 15%

T = time period = 3 years

Mark up (interest) $= \frac{P \times R \times T}{100}$

$= \frac{405000 \times 15 \times 3}{100}$

= Rs.182250

Balance = Markup + P

= Rs.(182250 + 405000)

= Rs.587250

Monthly installment $= \frac{587250}{3 \times 12}$

= Rs.16312.50

Total amount paid = Rs.(145000 + 587250) = Rs.73225

Example 16:

Mr. Akram got a truck on lease for 5 years through a bank. The price of truck is Rs.2000000. He paid 20% of price as down payment. The bank charged Rs.5000 as processing fee.

Find monthly installment and total amount paid if markup is 17%.

Solution:

Price of truck = Rs.2000000

Down payment = 20% of Rs.2000000

$= \frac{20}{100} \times$ Rs.2000000 = Rs.400000

P = Loan = Rs. (2000000 – 400000)

= Rs.1600000

T = 5 years, R = 17%

Markup $= \frac{P \times R \times T}{100}$

$= \frac{1600000 \times 17 \times 5}{100}$ = Rs.1360000

Monthly installment $= \frac{1360000}{5 \times 12}$ = Rs.22666.67

Total amount paid = Price + Processing fee + Markup

= Rs. (2000000 + 5000 + 1360000)

= Rs.3365000

Exercise 4.5

1. Yasmeen account balance is zero. She needs money to pay utility bills. She draws Rs.2000 through ATM. The bank charges Rs.6 daily as overdraft fee and a markup @ 7.5 % is also applied. Find the amount payable after 40 days.
2. Hanzala's account was overdrawn. He drew Rs.1000 through cheque. The daily overdraft fee is Rs.5. He has to pay an amount of Rs.2000 for electricity bill. He deposited Rs.3000 in his account after 30 days. Will he be able to pay the bill? Find the amount payable by him.
3. A motorcycle is sold on lease @ 12% for 3 years. Calculate monthly installment if sale price of motorcycle is Rs.60000.
4. Manahil bought a car worth Rs.600000 through a leasing company @ 16 % for 5 years. Find total amount paid by her in 5 years.

5. Mr. Shabbir purchased a car worth Rs.1250000 through a bank @ 18 % for 4 years. He agrees to pay an amount of Rs.400000 as down payment. Calculate monthly installment and total amount paid by him.

6. Mr. Aamir wants to get a car having price Rs.1500000 through a bank @ 17% for 7 years. He can pay Rs.150000 as down payment. Calculate monthly installment and total amount paid by him if the bank charges Rs. 5000 as processing charges.

Reading

4.3 PERCENTAGE

4.3.1 Profit and Loss

We have learnt in previous grades that

(i) Profit $= \text{S.P.} - \text{C.P.}$

(ii) Loss $= \text{C.P.} - \text{S.P.}$

(iii) Profit % $= \dfrac{\text{profit}}{\text{C.P.}} \times 100 = \dfrac{\text{S.P.} - \text{C.P.}}{\text{C.P.}} \times 100$

(iv) Loss % $= \dfrac{\text{loss}}{\text{C.P.}} \times 100 = \dfrac{\text{C.P.} - \text{S.P}}{\text{C.P.}} \times 100$

Moreover profit and loss percent are calculated on cost price.

4.3.2 Real Life Problems Involving Successive Transactions

We solve some examples to illustrate the method of successive transactions.

Example 17:

An article of C.P. Rs. 1000 is sold to a person A at a profit of 15%. After one month A sold that article to the person B at the profit of 5%.
Find the net profit and sale price of article.

Solution:

C.P. of article = Rs.1000, Profit percent = 15%

Profit gained by person A $= \dfrac{15}{100} \times 1000$

= Rs.150

$\therefore$ S.P. of article = Rs.1000 + Rs.150

= Rs.1150

This S.P. will become C.P. when article is sold again to B.

i.e. C.P. for next transaction = Rs. 1150

Profit percent = 5%

Profit gained by person B $= \frac{5}{100} \times 1150$

= Rs.57.50

Final S.P. of article = Rs.1150 + Rs.57.50

= Rs. 1207.50

Net profit = Rs.150 + Rs. 57.50

= Rs.207.50

Example 18:

Amir bought a house for Rs.2500000. After two years he sold it to Liaqat for Rs.2600000. Liaqat sold the house at the loss of 10%. Find final sale price and net profit or loss.

Solution:

C.P. of house = Rs.2500000

S.P. of house = Rs. 2600000

Profit = Rs.2600000 – Rs. 2500000

= Rs. 100000

The S.P. of house will be C.P. for the second transaction.

∴ C.P. of house = Rs.2600000

Loss percent = 10%

Loss on Rs. 2600000 $= \frac{10}{100} \times 2600000$

= Rs.260000

Final S.P. = Rs.2600000 – Rs.260000

= Rs.2340000

Net Loss = Rs.2500000 – Rs.2340000

= Rs.160000

4.3.3 Discount

Discount is a term which means deduction of some amount from the marked price given by the seller to customer / purchaser. Thus discount is the difference between the marked price and the sale price. i.e.

Discount = Marked Price – Sale Price

Note:
- Printed price of an article is also its marked price / listed price.
- If customers purchase any product on marked price without any discount, then this marked price will also be the sale price.

Relationship Among Marked Price, Sale Price and Discount

The relationship among discount, marked price (MP) and sale price (SP) is expressed by the following formulae.

- Discount $= MP - SP$
- Discount % $= \frac{\text{Discount}}{MP} \times 100$
- Discount $= \frac{MP \times \text{Discount }\%}{100}$
- SP $= MP \times \left(\frac{100 - \text{Discount }\%}{100}\right)$
- MP $= \frac{100 \times SP}{100 - \text{Discount }\%}$

Example 19:

A manufacturer allows 2.5% discount on an article costing Rs.12500. Find the amount of discount.

Solution:

MP of an article = Rs.12500

Discount = 2.5%

$$\text{Discount} = \frac{MP \times \text{Discount }\%}{100}$$

$$\text{Discount} = \text{Rs. } \frac{12500 \times 2.5}{100}$$

$$= \text{Rs. } \frac{12500 \times 25}{100 \times 10}$$

$$= \text{Rs. } 312.5$$

Example 20:

Marked price of a man's suit is Rs. 4800. Find its sale price if discount is 20 %.

Solution:

Marked price = MP = Rs. 4800, Discount = 20 %, SP = ?

$$SP = 4800 \times \left(\frac{100-20}{100}\right)$$

$$= 4800 \times \left(\frac{80}{100}\right) = \text{Rs. } 3840$$

Example 21:

What is the marked price of bicycle, which a retailer sells to a customer for Rs.8000 with 15 % discount?

Solution:

SP = Rs.8000, Discount = 15 %, MP = ?

$$MP = \frac{100 \times SP}{100 - \text{Discount \%}}$$

$$= \frac{100 \times 8000}{100-15}$$

$$= \frac{100 \times 8000}{85} = \frac{800000}{85}$$

$$MP = \text{Rs.}9411.76 \text{ (approx)}$$

Exercise 4.6

1. Anwar bought a shop for Rs.35000. He sold it to Akmal at a profit of 8%. Then Akmal sold the shop at a profit of 10%. Find net profit.
2. X bought an article for Rs.800. He sold it to Y at a profit of 10%. After three months Y sold that article at a loss of 10%. Find net profit or loss.
3. Ali bought some goods for Rs.1000 and sold them to Zain at a profit of Rs.100 then Zain also sold them and got the same profit. Which is better transaction?

4. The printed price of a 1.5 litre Pepsi bottle is Rs.80. Shopkeeper bought 3 dozen bottles with the discount of Rs.5 at each bottle. Calculate the discount % for 3 dozen bottles.

5. Sum of printed price of 10 notebooks is Rs.500 and shopkeeper gave a discount of Rs.50 to the customer. Calculate the discount % for one copy.

6. Supermarket gives 10 % discount on marked price of each item during a sale. Find the marked price and the amount of discount of the following if the sale price is (i) Rs.925 (ii) Rs.465

7. Find the marked price of the following, when
   (i) Discount = 12 %, SP = Rs.18000
   (ii) Discount = 7.5 %, SP = Rs.15000

Reading

4.4 INSURANCE

Insurance is an agreement between two parties. One party may be a person, a family or a company etc called insurer. Other party is an insurance company. In this agreement, the first party agrees to pay certain amount to the insurance company.

Insurance Policy

Insurance policy is written agreement between an insurer and an insurance company.

Premium

The amount paid by insurer in installments is called premium.

Maturity Period

The time period agreed by both the parties is known as maturity period.

4.4.1 Types of Insurance

There are two types of insurance.

(i) Life insurance (ii) Non – life (general) insurance

4.4.1(i) Life Insurance

In this type of insurance, a person called insurer agrees to pay premium during the maturity period. The insurance company pays back original amount and profit called bonus at the end of maturity period or on sudden death according to rules of the company.

Policy Fee

Each company gets some policy fee along with premium. For different companies, the policy fee is different. It is also different for different amount of premium.

Real Life Problems involving Life Insurance

Example 22:

Tariq got an insurance policy of Rs.75000. Calculate total premium paid by him @ 5% annually. Policy fee is 0.30 % of the total amount of policy and family income contract fee is Rs.525. Find the total amount paid by him in 20 years.

Solution: Maturity period = 20 years

Amount of policy = Rs.75000

First premium @ 5% = Rs.75000 × $\frac{5}{100}$

= Rs.3750

Policy fee = Rs.75000 × $\frac{0.30}{100}$

= Rs.225

Family income contract fee = Rs.525

Annual premium = Rs.(3750 + 225 + 525)

= Rs.4500

Total premium paid by Tariq = Rs.4500 × 20

= Rs.90000

> **Do You Know ?**
> There is difference between first (basic) and annual premium.

Example 23:

Lubna got an insurance policy worth Rs.100000. She pays annual premium Rs.5500 every year. What will she get at the end of 20 years if the company gives her the following bonuses? Maturity bonus @ 5% and profit @ 8% of policy amount per year.

Solution:

Maturity period = 20 years

Amount of policy = Rs.100000

Total premium paid by Lubna = Rs.5500 $\times$ 20 = Rs.110000

She will get the following amounts at the end of 20 years.

Policy amount = Rs.100000

Maturity bonus @ 5% for 20 years = Rs.$\frac{5}{100}\times 100000 \times 20$

= Rs.100000

Profit @ 8% for 20 years = Rs.$\frac{8}{100}\times 100000 \times 20$

= Rs.160000

Total amount got by her = Rs.(100000 + 100000 + 160000)

= Rs.360000

Example 24:

An insurer got a policy of Rs.100000 for a maturity period of 20 years. Amount of annual premium is Rs.5800. Find amount paid by him in 5 years. What will his family get if he died 5 years after getting the policy where rate of bonus is 5% along with an additional amount of Rs.4500 per year for the remaining period?

Solution:

Maturity period = 20 years

Amount of policy = Rs.100000

Annual premium = Rs.5800

Amount paid by insurer for 5 years = Rs.5800 $\times$ 5

= Rs.29000

After death, his family will get policy amount = Rs.100000

Bonus @ 5% for 15 years = Rs.100000 $\times \frac{5}{100}\times 15$

= Rs.75000

Additional amount for the rest of 15 years = Rs.4500 $\times$ 15

= Rs.67500

Total amount got by his family = Rs.(100000+75000 + 67500)

= Rs. 242500

4.4.1(ii) Vehicle Insurance

People or companies also get insurance policies against vehicles called vehicle insurance.

In this way they try to cover the risks of fire,theft, accident etc. Rates of premium for vehicles are different for different amounts and for different time span. A person or company can get insurance policy on the total price or partial price of vehicles. As long as the price value of vehicles decreases, the value of annual premium is also decreased.

Depreciation:

Depreciation is the decrease in the value of a hard asset (vehicle or property) over a period of time.Usually the depreciation in the value is taken as 10%.

Service Charges:

Just like the policy fee in the case of life insurance, the companies also charge some fee from the insurer called service charges.

Real Life Problems involving Vehicle Insurance

To understand vehicle insurance, we solve some examples.

Example 25:

Akmal got an insurance policy for his car @ 4% for 2 years. The price value of car is Rs.900000. Find total premium paid by him if he did not claim for any damage during maturity period. The rate of depreciation is 10 %.

Solution:

Maturity period = 2 years

Price value of car = Rs.900000

Rate of premium = 4%

First premium @ 4% $= \text{Rs.}900000 \times \frac{4}{100}$

$= \text{Rs. } 36000$

Depreciation @ 10% $= \text{Rs.}900000 \times \frac{10}{100}$

$= \text{Rs.}90000$

Depreciated price $= \text{Rs.}900000 - \text{Rs.}90000$

$= \text{Rs.}810000$

Second premium @ 4% $= \text{Rs.}810000 \times \frac{4}{100}$

$= \text{Rs.}32400$

Total premium paid $= \text{Rs.}\ (36000 + 32400)$

$= \text{Rs.}68400$

Exercise 4.7

1. Calculate annual premium paid by insurer to the company to get a policy of Rs.200000 for a maturity period of 20 years if first premium is 4.5 % of policy amount and policy fee is Rs.350.
2. Calculate total amount paid by an insurer to the company to get a policyof Rs.100000 for a maturity of 20 years if first premium is 5% of policy amount and policy fee is Rs.300.
3. Saad got a policy worth Rs.50000 along with policy fee Rs.200. The rate of first premium is 4.8% of policy amount. Calculate half yearly premium @ 52%, quarterly premium @ 27% and monthly premium @ 9% of annual premium.
4. Amount of policy got by Rida is Rs.150000 for 25 years. Calculate amount received by her from the company after maturity period if

   Bonus = 0.045 of policy amount for 25 years
   Family income bonus = Rs.100000
   Maturity bonus = 2.5% of policy amount for 25 years
5. An insurer got a policy worth Rs.100000 for 20 years and died after 2 years. Calculate total amount given by the company to his family.

   If bonus = Rs.6000 per year for the rest period

   Additional amount = Rs.5000 per year for the rest period.
6. Annual premium paid by a person is Rs.7750 @ 5% where policy fee is Rs.250.Find the amount of policy.

7. A person bought an insurance policy against his car worth Rs.1200000 @ 5% for 2 years. Find the total premium paid by him. Rate of depreciation is 10%.

8. Akbar bought a car worth Rs.500000. He got it insured @ 3.5% for 5 years. After two years his car destroyed due to an accident. How much loss of Akbar was recovered? Rate of depreciation is 10%.

9. Asma got a vehicle insurance policy worth Rs.300000 @ 4.5% for 6 years. After 2 years, she claimed for Rs.100000. How much benefit did she's get? Rate of depreciation is 10%.

10. A company insured a truck @ 4% for 10 years and paid Rs.50000 as first premium. Find the amount of policy. Also find second premium. Rate of deprecation is 10%.

Reading

4.5 INCOME TAX

Income Tax and Exempt Amount

Income tax is imposed on the persons by the government whose income exceeds a certain limit during one year called fixed amount. This fixed amount is called **rebate** or **relief**. Thus rebate is income for which tax is 0 %.

Rates of Income Tax for the Year 2012-13

Tax Year 2012-13 (Salary)	
Up to Rs. 400000	0 %
Rs.400001 to Rs.750000	5 % of amount exceeding 400000
Rs.750001 to Rs.1500000	Rs.17500 + 10% of amount exceeding Rs.750000
Rs.1500001 to Rs.2000000	Rs.95000 + 15% of amount exceeding Rs.1500000
Rs.2000001 to Rs.2500000	Rs.175000 + 17.5% of amount exceeding Rs.2000000
Rs.2500001 and above	Rs.420000 + 20% of amount exceeding Rs.2500000

Tax Year 2012-13 (Business)	
Up to Rs. 400000	**0 %**

Rs.400001 to Rs.750000	**10 % of amount exceeding 400000**
Rs.750001 to Rs.1500000	**Rs.35000 + 15% of amount exceeding Rs.750000**
Rs.1500001 to Rs.2500000	**Rs.147500 + 20% of amount exceeding Rs.1500000**
Rs.2500001 and above	Rs.347500 + 25% of amount exceeding Rs.2500000

Example 26:

Salary of Ms. Azra is Rs.54000. Find the amount of income tax @ 6% if she paid Rs.3300 as Zakat and Rs.2000 as wealth tax. The rebate on the tax is Rs.400000.

Note:
(a) A full time teacher or researcher is allowed to a reduction in tax @ 75% tax payable on his income from salary.
(b) A reduction of 50% of the tax payable is allowed to a senior citizen if the age of taxpayer is 60 years or more and taxable income does not exceed Rs.400000. Such taxpayer is called senior taxpayer.
(c) Tax is calculated on annual income not on monthly income.

Solution:

Salary = Rs.54000

Annual income = Rs.54000 × 12

= Rs.648000

Zakat = Rs.3300

Wealth tax = Rs.2000

Net income = Rs.648000 – Rs. (3300 + 2000)

Rs.648000 – Rs.5300

= Rs.642700

Rebate = Rs. 400000

Taxable income = Rs. (642700 – 400000)

= Rs. 242700

Income tax @ 6% = $\text{Rs.}242700 \times \frac{6}{100}$

= Rs.14562

Zakat is calculated on the savings after one year while income tax is calculated on the gross income.

Example 27:

Mr. Jameel is full time teacher. His salary is Rs.40000. Calculate income tax if there is 75% reduction being full time teacher on the tax. The rate of income tax is 7.5 %. The rebate on the tax is Rs.300000.

Solution:

Salary = Rs.40000

Annual income = Rs.40000 × 12 = Rs.480000

Rebate = Rs.300000

Taxable income = Rs. (480000 – 300000)

= Rs.148000

Income tax @ 7.5% = Rs.148000 × $\frac{7.5}{100}$

= Rs.11100

75% reduction on the tax = Rs.11100 × $\frac{75}{100}$

= Rs.8325

Amount of tax = Rs. (11100 – 8325)

= Rs.2775

Example 28:

Calculate the taxable income and income tax @ 4.5 % of an individual who has provided the following information:

Income from property = Rs.120000

Income from business = Rs.340000

Salary = Rs.285000

Tax rebate = Rs.400000

Solution:

Total income = Rs.120000 + Rs.340000 + Rs.285000

= Rs.745000

Taxable income = Rs.645000 – Rs.400000

= Rs.245000

Income tax @ 4.5 % = Rs.245000 × $\frac{4.5}{100}$

= Rs.11025

Exercise 4.8

1. Ms. Haleema provided the following information for the tax year.

Salary	=	Rs.34400 per month
Income from business	=	Rs.22000 per month

Calculate her income tax. @ 4.50 %.The rebate on the tax is Rs.400000.

2. Mr. Iftikhar is a professor in a government college. Compute income tax paid by him @ 6 % if his monthly income is Rs.50000. The reduction in the tax is 75% and the rebate on the tax is Rs.400000.

3. Mr. Manzoor Qadir provided the following information for the tax year.

Salary income	=	Rs.232000
Rent from house	=	Rs.104400
Income from agricuture	=	Rs.36000

Calculate his income tax if the rebate on the tax is Rs.200000 and rate of tax is 5%.

4. Mr. Asif is a doctor and runs his private clinic. His payment account for the tax year is:

Consultation fee	=	Rs.862000,
Gifts from patients	=	Rs.42000

Calculate his income tax if he gives Rs.48000 as rent of clinic and Rs.60000 as salary to his assistant. (Rate of tax is 4 % and rebate on the tax is Rs.400000.)

(**Hint**:The rent of clinic and salary of assistant is not taxable)

5. The following information is available in respect of Mr. Ahmed Khan for the current tax year.

Salary income	=	Rs.140000
Bonus	=	Rs.30000
Business income	=	Rs.290000
Royalty	=	Rs.40000

Calculate tax @ 10%. The rebate on the tax is Rs.200000.

Review Exercise 4

1. Encircle the correct answer in the following.

i. Which of the following is a type of bank account?
(a) ATM (b) current account (c) pay order (d) cheque

ii. Which of the following is a negotiable instrument?
(a) demand draft (b) debt card (c) credit card (d) PLS

iii. Online banking is also called __________ banking.
(a) commercial (b) internet (c) conventional (d) fast

iv. What can we do through an ATM card?
(a) deposit cash (b) transfer money (c) pay bills (d) all a, b & c

v. If 1 Saudi Riyal is of Rs.35, then how many Saudi Riyals can be purchased for Rs.1400?
(a) Rs.40 (b) SAR.41 (c) SAR.40 (d) SAR.45

vi. Rashida deposited Rs.15000 in her saving account. She got a profit of Rs.3000 after one year. What is the rate of profit?
(a) 2 % (b) 10 % (c) 20 % (d) 30 %

vii. What will be the markup for the principal of Rs.100 @ 10 % in 10 years?
(a) Rs.100 (b) Rs.10000 (c) Rs.1000 (d) Rs.10

viii. Usually rate of depreciation for vehicle is ____________.
(a) 8 % (b) 10 % (c) 20 % (d) 50%

ix. The amount payable at the time of agreement for leasing a car is called _____.
(a) processing fee (b) installment (c) down payment (d) markup

x. Share of father and mother is _______ of the property.
(a) $\frac{1}{6}$ (b) $\frac{1}{4}$ (c) $\frac{1}{8}$ (d) $\frac{1}{2}$

xi. If a deceased man has only one daughter, she will get____of the property.
(a) $\frac{1}{3}$ (b) $\frac{1}{2}$ (c) $\frac{2}{3}$ (d) $\frac{1}{6}$

xii. A coloured television set costing Rs.25000 was purchased for Rs.24500. What is the rate of discount?

(a) 3% (b) 2.5% (c) 2% (d) 4%

xiii. Gross income is the total income earned by a person during one:

(a) day (b) Week (c) month (d) year

xiv. Year starting from first July is called:

(a) new year (b) financial year (c) Lunar year (d) Solar year

xv. Income computed after deduction of Zakat and welfare fund etc. is

(a) ta*x*able income (b) net income

(c) gross income (d) personal income

2. Amna has Rs.90000 in her account. She withdraws an amount of Rs.60000 by using demand draft. The bank deducts service charges @ 0.5 % on the deduction. Find services charges and the balance in the account. What will be the Zakat on the balance?.

3. Amount of policy got by a person is Rs.200000 @ 5 % for 20 years. Family income contract = 1% of policy amount. Calculate

(i) Amount paid by him to the company.

(ii) Amount received by him from the company after maturity period if Bonus = 6 % of policy amount for 20 years

4. Rashida got an insurance policy worth Rs.500000 for her car @ 4.5% for 5 years. After 2 years, she claimed for Rs.180000. How much benefit did she's get? Rate of depreciation is 10%.

5. 33 men can do two third of a work in 12 days working 10 hours daily. How much time is required daily to complete remaining work in 6 days if 3 men are not available now?

6. A contractor got a contract to construct a bridge in 96 days. He employed 40 workers. After 60 days $\frac{2}{5}$ work was completed. How many more workers are required to complete the work in specified time?

7. Rehan and Saeed started a business with Rs.50000 and Rs.40000 respectively. After 6 months Saeed took his money back and Asim joined them with an investment of Rs.70000. They got a profit of Rs.63000 at the end of the year. Find the share of each.

8. A woman died leaving the following property.
Bank balance = Rs.430000, Gold 6 tolas = Rs.360000.
Divide her property among her son and her husband if two tola gold was given to a poor family.

9. Amna bought 15 kg rice at 5% discount for Rs.1500, 10 kg sugar at 4% discount for Rs.600 and 30 kg flour at a discount of 5% for Rs.1200. Find
   i. marked price of each item
   ii. discount for each item
   iii. total discount.

Key Point

- The combination of two or more proportions (either direct or inverse or both) is called compound proportion.
- A partnership is a business run by at least two persons, companies or parties in which amount of investment and time span may be different.
- Inheritance is an important branch of the family laws of the Muslims. After the death of a person, most of his rights are transferred to his legal heirs.
- An institution which deals in money is called a bank.
- The commercial bank deposits are of four types. Current deposit accounts, PLS saving accounts, PLS term deposit accounts and foreign currency accounts.
- A document which enables the person holding it, to get a specified sum of money from the bank is called negotiable instrument. The negotiable instruments are of three types: cheque, demand draft and pay order.
- Online banking is also called internet banking. It is used to perform transactions over the internet through a bank or credit union etc.
- Auto Teller Machine (ATM) is a computerized telecommunication device. It facilitates customers of a bank to make transactions any time.
- When we deposit money in a bank for a certain period of time, the bank gives us some extra amount called profit or markup.
- If the principal amount (P) is deposited or borrowed at the rate(R%) per annum for T years then the profit/markup can be calculated by the formula

$$\text{Profit/Markup} = \frac{\text{PRT}}{100}$$

- Insurance policy is written agreement between an insurer and an insurance company stating the obligations and responsibilities of each party.
- The amount paid by an insurer in installments is called premium.

- The time period agreed by both the parties is known as maturity period.
- There are two types of insurance life insurance and non – life (general) insurance.
- Lease is an agreement between two parties giving one party the right to use the asset (motor vehicles, houses etc) for a period of time.
- **The amount payable by the customer to the owner at the time of agreement is called down payment.**
- Vehicle insurance is insurance purchased for road vehicles.
- Overdraft, running finance and demand finance are special types of loan provided by the banks to their account holders. The account holders can draw a limited amount from their accounts even they have zero balance after paying some extra amount.
- Gross income is the total income earned by a person during one year.
- Net income for salaried person is an income given to him after deduction of G.P fund, benevolent fund etc. during one year.
- Income tax is imposed on the persons whose income exceeds a certain limit during one year called fixed amount.

5

POLYNOMIALS

This is 09 days unit (periods including homework)

After studying this unit, you will be able to:

- ❖ recall constant, variable, literal and algebraic expression.
- ❖ define polynomial, degree of polynomial and coefficient of polynomial.
- ❖ recognize polynomials in one, two or more variables.
- ❖ recognize polynomials of various degrees (e.g. linear, quadratic, cubic and higher order polynomials)
- ❖ add, subtract and multiply polynomials.
- ❖ divide a polynomial by a linear polynomial.

Reading

INTRODUCTION

We have already studied in the Grade-VII about constant, variable, algebraic expression, polynomials, addition and subtraction in algebraic expression.

Now we will revisit the algebraic expression and polynomials for strengthening the conceptual understanding.

Constants

A symbol which has a fixed numerical value is called a constant or a quantity that does not change its value in a given expression or equation is called a constant. e.g. 2, 6, 0, -3, etc, are constants. In the expression $3x + 4$, 3 and 4 are constants.

Variables

A quantity which has no fixed value but takes various numerical values is called a variable. For example the height of a student is a variable, as it varies from student to student. A variable is denoted by letter like x, y, z, u, v etc. A combination of constant and a variable is also a **variable**.

(i) In $x + y + 12$, the two numbers x and y are variables but 12 is a constant.

(ii) In $7 + y = 15$, y is a variable and 7 and 15 are constants.

(iii) In $x^2 + y + z$, x, y and z all are variables.

Literal

In arithmetic, we use numbers 1, 2, 3, 4. Each of them has a definite value, whereas in algebra in addition to numbers, letters such as a, b, c, d... z are used. The letters used in algebra may represent any number in general.

For example, in arithmetic $5 + 4 = 9$, means that the sum of 5 and 4 is equal to 9. In algebra $x + y = z$ means that the sum of two numbers represented by x and y is equal to the number represented by z.

Similarly, if $x + y = 12$, then x and y may stand for any pair of numbers whose sum is 12. For example 7 and 5, 9 and 3, as $7 + 5 = 12$ and $9 + 3 = 12$.

5.1 ALGEBRAIC EXPRESSIONS

A statement in which variables or constants or both are connected by arithmetic operations (i.e. + , – , ×, ÷) is called an algebraic expression.

For example,

$10x$, $3(a + b) - 4$, 0, -5

$x - \sqrt{2}y$, $\frac{1}{x}$, $\sqrt{b^2 - 4ac}$ etc.

Note: Components of an algebraic expression are:
- Numbers
- Signs of operations (+ , – , ×, ÷)
- English alphabets

5.2 POLYNOMIALS

Polynomials are algebraic expressions consisting of one or more terms in which exponents of the variables involved are non negative (either zero or positive) integers.

For example,

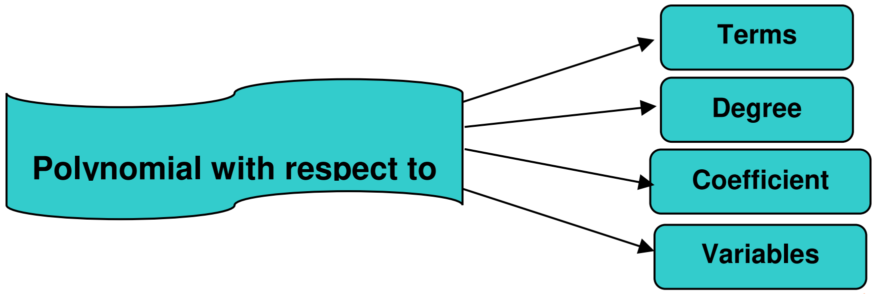
$5x$, $5x^2 - xy + y^2$, $a^2 + ab + 7$, $p^2 + pq + 8$, $9xy^2 + 6xy + 10$,

$x^2 - \frac{4}{3}x^3 + 6x + \frac{1}{4}$, 0, $\frac{3}{4}x + \frac{3}{4}y^2z$, $-\sqrt{\frac{3}{9}}y^3$, $\sqrt{2}x^4 - \pi x^2 - \sqrt{10}$ etc.

All these are example of polynomials.

The expressions x^{-3}, $y^2 + \frac{1}{y^2}$, $\sqrt[3]{y^4}$, $2y^{\frac{1}{2}}$ are not polynomials because their exponents are not non negative integers (whole numbers).

5.3 CLASSIFICATION OF POLYNOMIALS



(a) Polynomials w.r.t. Terms

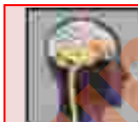
- A polynomial having one term is called a monomial. e.g., 3, x , 0 etc.
- A polynomial having two terms is called a binomial.

e.g. a + b, 2 x + y, 7ab – $\frac{1}{3}$, x^3 + 2$\frac{1}{4}$ etc.

- A polynomial having three terms is called a trinomial.

e.g. $x^2 + 2xy + y^2$, $\frac{1}{8}x^3 + \frac{3}{4}x^2 - 9$ etc.

All other polynomials having more than three non-zero terms are called polynomials of four terms, five terms, six terms and so on.



Plus the Memory

When a polynomial is written in descending order, then the first coefficient is called the leading coefficient.
e.g. in p(x) = $3x^4 - 2x^2 + 1$, the leading coefficient is 3.
Here p(x) means that the variable in the polynomial is 'x'.

(b) Polynomials w.r.t. Degree

- **Zero Polynomial or No Degree Polynomial**

'0' is called a polynomial of no degree. Also $0x^3 + 0x$ is a no degree polynomial, because coefficients are always zero in zero polynomial.

- **Constant Polynomial**

A polynomial having degree zero is called a constant polynomial.

e.g. 2, –5, $\frac{1}{2}$, $\sqrt{5}$ are all constant polynomials.

- **Linear polynomial**

A polynomial having degree one is called a linear polynomial.

e.g. x, $2x - y$, $-7xy^0$ etc.

- **Quadratic polynomial**

A polynomial having degree two is called a quadratic polynomial.

e.g. $2x^2 + 7$, ax + 2xy + 3, $-\frac{3}{4}xyz^0$ etc.

The highest exponent of the variable involved in a polynomial is called its degree. If more than one variable are being multiplied in the terms of a polynomial, then the degree of that polynomial is the highest sum of exponents of variables being multiplied.

- **Cubic polynomial:**

A polynomial having degree three is called a cubic polynomial.

e.g. $9x^3 - 7x + 5$, $-9xzy$, $3x^2y - \frac{3}{4}z$ etc.

All other polynomials have no specific name w.r.t. degree but simply, we call them polynomials of degree four, degree five and so on.

(c) Polynomials w.r.t. Coefficients

Polynomials can also be classified according to the coefficients of the variables involved in each term of them. e.g. $-3x^2 + 4x - 1$ is a polynomial w.r.t. integers because $-3, 4, -1 \in Z$. Similarly, $\frac{1}{2}y^3 - \pi x^2 y + \frac{22}{7}$ is a polynomial w.r.t. real numbers because $\frac{1}{2}, -\pi, \frac{22}{7} \in R$. Just like this we can also check the coefficients in any polynomial and classify it accordingly.

(d) Polynomials w.r.t. Variables

Classification of polynomials is also possible with respect to the variables involved in them. e.g. $x^2y - 7$ is a polynomial in two variables x, y. We can express it as

$$P(x, y) = x^2y - 7$$

Similarly $6x^2yz + 12xy^6z^6 + 9z4$ is a polynomial in three variables x, y and z. We may write it as $Q(x, y, z) = 6x^2yz + 12xy^6z^6 + 9z^4$.

Exercise 5.1

1. Which of the following algebraic expressions are polynomials?

(i) $6y + 9$ (ii) $x^2 + 2x + \frac{7}{x}$ (iii) $6y + \frac{1}{y}$

(iv) $x^{\frac{3}{2}} + 3x + 2$ (v) $x^2 + \frac{x}{4}$ (vi) $y^5 - 2 + \frac{y^3}{y^2}$

2. Write the degree of the following polynomials and also recognize number of variables.

(i) $9x + 7$ (ii) $6x^2 - 3x + 2$ (iii) 7

(iv) $10x^4 + 6x^2 + 10$ (v) $x^3 - 10xy^2$ (vi) $6x + 7xy + 12$

(vii) $y^2 - 6x^2z^3 + y$ **(viii)** $x^2yz + 6y^4z + x^3z^2$

3. Which of the following are linear, quadratic and cubic polynomials.

(i) $9x + 7$ **(ii)** $6x^2 - 3x + 2$ **(iii)** $2x^3 + 2x^2 - 4$

(iv) $x + 7x^2 + 1$ **(v)** $x^3 - 6x$ **(vi)** x

Reading

5.4 ADDITION AND SUBTRACTION OF POLYNOMIALS

We have already studied about addition and subtraction of polynomials in Grade-VII, but in this class we will learn addition and subtraction of polynomial of higher degree.

An algebraic expression consists of like and unlike terms, while adding and subtracting we collect and add the like terms. The sum of several like terms is another term whose coefficient is the sum of the coefficients of those like terms. We add and subtract only like terms.

The sum of two positive terms is positive, the sum of two negative terms is negative and the sign of greater term is taken, when the two terms have different signs.

Example 1:

Find the sum of $8x^3 - x^2 + 3$ and $x^4 + 4x^3 - x$.

Solution:

$[8x^3 - x^2 + 3] + [x^4 + 4x^3 - x]$

$= x^4 + 8x^3 + 4x^3 - x^2 - x + 3$

$= x^4 + 12x^3 - x^2 - x + 3$

Example 2:

Find the sum of $7x^4 + 6x^3 - 6x + 3$ and $-5x^4 - 3x^3 + 8x - 5$.

Solution:

$[7x^4 + 6x^3 - 6x + 3] + [-5x^4 - 3x^3 + 8x - 5]$

$= 7x^4 + 6x^3 - 6x + 3 - 5x^4 - 3x^3 + 8x - 5$

$= (7x^4 - 5x^4) + (6x^3 - 3x^3) + (-6x + 8x) + (3 - 5)$

$= 2x^4 + 3x^3 + 2x + (-2)$

$= 2x^4 + 3x^3 + 2x - 2$

Example 3:

(i) Subtract $4x^2$ from $-6x^2$

(ii) Subtract $3x^2 - 6x + 5$ from $6x^2 - 10x - 8$

(iii) Subtract $4x^4 - 5x^3 + 6x^2 - 8$ from $8x^4 + 9x^3 - 10x^2 + 10$

Solution:

(i) $(-6x^2) - (4x^2)$

$= -6x^2 - 4x^2$

$= -10x^2$

(ii) $(6x^2 - 10x - 8) - (3x^2 - 6x + 5)$

$= 6x^2 - 10x - 8 - 3x^2 + 6x - 5$

$= 6x^2 - 3x^2 - 10x + 6x - 8 - 5$

$= 3x^2 - 4x - 13$

(iii) $(8x^4 + 9x^3 - 10x^2 + 10) - (4x^4 - 5x^3 + 6x^2 - 8)$

$= 8x^4 + 9x^3 - 10x^2 + 10 - 4x^4 + 5x^3 - 6x^2 + 8$

$= 8x^4 - 4x^4 + 9x^3 + 5x^3 - 10x^2 - 6x^2 + 10 + 8$

$= 4x^4 + 14x^3 - 16x^2 + 18$

Exercise 5.2

1. Find the sum of following polynomials.

(i) $6x^4 + 10x^2 + 5,\ 8x^4 - 7x^2 - 3$

(ii) $5x - 6x^2 + x^3,\ -8x + 6x^2 - 10x^3 + x^4$

(iii) $2x^4 - 2x^3 + 6x - 1,\ 3x^4 - 4x^3 - 5x + 3$

(iv) $7x^4 - 6x^3 + 10x - 9,\ 6x^4 + 10x^3 - 7x + 11$

(v) $10x^4 - 10x^3 + 5x^2 - 6x + 3,\ 5x^4 + 8x^3 - 3x^2 + 7x - 10$

2. If $A = 4x^3 - 3y^2 + 5z$

$B = 6x^3 - 5y^2 + 8z$

$C = -2x^3 + 6y^2 - 9z$

then find $A + B + C$

3. If $P = 3x^2 + 4xy - 4y^2$

$Q = x^2 - 3xy + 5y^2$

$R = -4x^2 + xy + y^2$

then find $P + 2Q - R$

4. Subtract

(i)	$6x^4 + 3x^2 - x + 1$	from	$8x^4 + 4x^2 + x - 1$
(ii)	$x^4 + 9x^3 + 3x + 2$	from	$x^4 + 7x^3 + 2x + 9$
(iii)	$2y^3 + 5y^2 - 9y - 10$	from	$4y^3 + 9y^2 + 6y + 13$

5. The sum of two quantities are $4x^3 + 6x^2y^2 + 9y^3$. If one of them is $2x^3 + 6x^2y^2 + 6y^3$. Find the other.

6. If the two sides of a triangle are $x - y + 4z$ and $x + 2y - 5z$. Find the third side when the perimeter is $3x + 2y + z$.

Reading

5.5 MULTIPLYING TWO POLYNOMIALS OF DEGREE

The product of two polynomials is obtained by the application of the laws of exponents. Detail has been already studied in Grade-VII.

Example 4: Simplify the following.

(i) $3x^2 \times 4x^3 = (3 \times 4)\, x^{2+3}$
$= 12x^5$

(ii) $-15x^3 \times 6x^4 = (-15 \times 6)x^{3+4}$
$= -90x^7$

(iii) $-10x^2y^2 \times 15xy^2 = (-10 \times 15)x^{2+1}\, y^{2+2}$
$= -150x^3y^4$

(iv) $x^3y^2\,(x^2 + y^2) = x^{3+2}y^2 + y^{2+2}x^3$
$= x^5y^2 + y^4x^3$

While multiplying the following rules may be adopted.

(i) Write the product of the numerical coefficients.

(ii) All the different bases occurring in multiplication of several terms are to be taken in the product with exponents equal to the sum of the exponents of the like bases in the terms.

(iii) The sign of the product is minus, if one term is positive and other term is negative.

(iv) The sign of the product is plus, if both the terms have the same signs(either both positive or both negative).

Example 5:

Find the product of

(i) $(x^2 + 3x)$ and $(x^2 + 4)$ **(ii)** $(x^2 - 2x)$ and $(x - 4)$

(iii) $(x^2 - 4x + 7)$ and $(3x^2 - 4x + 6)$ **(iv)** $(x - y)$ and $(x^2 + xy + y^2)$

(v) $(a^2 - 3)$ and $(5a^4 - 6a^2 - 7)$

(vi) $(x^4 + x^3 - 7x^2 + 3)$ and $(x^4 - 2x^3 - 9x + 4)$

Solution:

(i) $(x^2 + 3x)(x^2 + 4) = x^2(x^2 + 4) + 3x(x^2 + 4)$

$= x^4 + 4x^2 + 3x^3 + 12x$

$= x^4 + 3x^3 + 4x^2 + 12x$

(ii) $(x^2 - 2x)(x - 4) = x^2(x - 4) - 2x(x - 4)$

$= x^3 - 4x^2 - 2x^2 + 8x$

$= x^3 - 6x^2 + 8x$

(iii) $(x^2 - 4x + 7)(3x^2 - 4x + 6)$

$= x^2(3x^2 - 4x + 6) - 4x(3x^2 - 4x + 6) + 7(3x^2 - 4x + 6)$

$= 3x^4 - 4x^3 + 6x^2 - 12x^3 + 16x^2 - 24x + 21x^2 - 28x + 42$

$= 3x^4 - 16x^3 + 43x^2 - 52x + 42$

(iv) $(x - y)(x^2 + xy + y^2)$

$= x(x^2 + xy + y^2) - y(x^2 + xy + y^2)$

$= x^3 + x^2y + xy^2 - x^2y - xy^2 - y^3 = x^3 - y^3$

(v) $(a^2 - 3)(5a^4 - 6a^2 - 7)$

$= a^2(5a^4 - 6a^2 - 7) - 3(5a^4 - 6a^2 - 7)$

$= 5a^6 - 6a^4 - 7a^2 - 15a^4 + 18a^2 + 21$

$= 5a^6 - 21a^4 + 11a^2 + 21$

(vi) $(x^4 + x^3 - 7x^2 + 3)(x^4 - 2x^3 - 9x + 4)$

$= x^4(x^4 - 2x^3 - 9x + 4) + x^3(x^4 - 2x^3 - 9x + 4)$

$- 7x^2(x^4 - 2x^3 - 9x + 4) + 3(x^4 - 2x^3 - 9x + 4)$

$= x^8 - 2x^7 - 9x^5 + 4x^4 + x^7 - 2x^6 - 9x^4 + 4x^3 - 7x^6 + 14x^5$

$+ 63x^3 - 28x^2 + 3x^4 - 6x^3 - 27x + 12$

$= x^8 - x^7 - 9x^6 + 5x^5 - 2x^4 + 61x^3 - 28x^2 - 27x + 12$

Exercise 5.3

Evaluate.

1. $(x - 4)(x^2 - 5x - 6)$
2. $(a + b)(a^2 - ab + b^2)$
3. $(2x - 1)(x^2 - 3x - 2)$
4. $(x^2 + y^2)(x^4 - x^2y^2 + y^4)$
5. $(x^2 + xy + y^2)(x^2 - xy + y^2)$
6. $(x^2 + 5x + 4)(x^2 - 2x - 6)$
7. $(3y^4 - 2y^2 + 4y - 5)(y^2 + 1)$
8. $(4x^3 + 6x^2 - 4x + 3)(3x^3 - 4x + 3)$

Simplify the following.

9. $(x^2 + 2x - 1)(x - 1) + (x^2 - 2x + 1)(x + 1)$
10. $(y^2 + 5y)(y - 1) - (y^2 + 2y)(y - 1)$
11. $(5x - 1)(6x + 7) + (7x - 2)(2x + 5)$

Reading

5.6 DIVISION OF A POLYNOMIAL BY A LINEAR POLYNOMIAL

Consider the following example.

$$\frac{48}{4} = \frac{2\times2\times2\times2\times3}{2\times2} = 12$$

Similarly consider the following examples in algebra.

(i) $\frac{x^3}{x}$

$$\frac{x^3}{x} = \frac{x.x.x}{x}$$

$= x \,.\, x$

$= x^2$

(ii) $\frac{15x^5}{3x^2}$

$$= \frac{15x^2}{3x^2} \cdot \frac{x.x.x.x.x}{x.x}$$

$= 5x^3$

(iii) $\frac{25x^6}{5x^2} = \frac{{}^{5}25}{5}\ \frac{x^6}{x^2}$

$= 5x^{6-2}$

$= 5x^4$

(iv) $\frac{5y^{10}}{3y^7} = \frac{5}{3}\ y^{10-7}$

$= \frac{5}{3}y^3$

(v) $\frac{-21x^5y^2}{3x^3y}$

(vi) $(x^5 - x^2) \div x^2$

$= -3x^{5-3}y^{2-1}$

$= -3x^2y$

$= \dfrac{x^5 - x^2}{x^2}$

$= \dfrac{x^5}{x^2} - \dfrac{x^2}{x^2}$

$= x^3 - 1$

This method is known as “short division method”.
The above examples may also be solved by the method of long division.

(i)

$$\begin{array}{r|l} & x^2 \\ \hline x & x^3 \\ & \pm x^3 \\ \hline & 0 \end{array}$$

(ii)

$$\begin{array}{r|l} & 5x^4 \\ \hline 3x & 15x^5 \\ & \pm 15x^5 \\ \hline & 0 \end{array}$$

(iii)

$$\begin{array}{r|l} & 5x^5 \\ \hline 5x & 25x^6 \\ & \pm 25x^6 \\ \hline & 0 \end{array}$$

(iv)

$$\begin{array}{r|l} & 5y^9 \\ \hline 3y & 5y^{10} \\ & \pm 5y^{10} \\ \hline & 0 \end{array}$$

(v)

$$\begin{array}{r|l} & -3x^2y \\ \hline 7x^3y & -21x^5y^2 \\ & \mp 21x^5y^2 \\ \hline & 0 \end{array}$$

Note: We prefer method of long division when divisor consists of two or more terms.

Example 6:

Divide $x^2 + 2xy + y^2$ by $x + y$.

Solution:

$$\begin{array}{r|l} & x + y \\ \hline x + y & x^2 + 2xy + y^2 \\ & \pm x^2 \pm xy \\ \hline & \quad xy + y^2 \\ & \quad \pm xy \pm y^2 \\ \hline & \quad 0 \end{array}$$

Description of steps

For division the following rules may be adopted.

- **(a)** The coefficients are to be divided.
- **(b)** The difference of exponents of like variables is taken.
- **(c)** The sign of the terms is changed when subtracted.
- **(d)** In first step write both the polynomials in descending order.

(e) Divide the first term of the dividend by the first term of divisor (the coefficients are to be divided).

(f) Multiply the divisor by the quotient got in (b) (the difference of exponents of like variables is taken).

(g) Subtract the product from the dividend. (The sign of the term is changed when subtracted).

(h) When the remainder is zero, the division is completed.

Example 7:

Divide $x^2 + 3x + 2$ by $x + 1$.

Solution:

$$
\begin{array}{r|l}
 & x+2 \\
\hline
x+1 & x^2+3x+2 \\
 & \underline{\pm x^2 \pm x} \\
 & \quad 2x+2 \\
 & \quad \underline{\pm 2x \pm 2} \\
 & \qquad 0
\end{array}
$$

Example 8:

Divide $x^2 - 7x + 10$ by $x - 5$.

Solution:

$$
\begin{array}{r|l}
 & x-2 \\
\hline
x-5 & x^2-7x+10 \\
 & \underline{\pm x^2 \mp 5x} \\
 & \quad -2x+10 \\
 & \quad \underline{\mp 2x \pm 10} \\
 & \qquad 0
\end{array}
$$

Example 9:

Divide $x^3 + 2x - 12$ by $x - 2$.

Solution:

$$
\begin{array}{r|l}
 & x^2+2x+6 \\
\hline
x-2 & x^3+2x-12 \\
 & \underline{\pm x^3 \qquad \mp 2x^2} \\
 & \quad 2x^2+2x-12 \\
 & \quad \underline{\pm 2x^2 \mp 4x} \\
 & \qquad 6x-12 \\
 & \qquad \underline{\pm 6x \mp 12} \\
 & \qquad\quad 0
\end{array}
$$

Example 10: Divide $24x^2 + 2xy - 35y^2$ by $4x + 5y$.

Solution:

$$
\begin{array}{r|l}
 & 6x - 7y \\
\hline
4x + 5y & 24x^2 + 2xy - 35y^2 \\
 & \pm 24x^2 \pm 30xy \\
\hline
 & \quad -28xy - 35y^2 \\
 & \quad \mp 28xy \mp 35y^2 \\
\hline
 & \qquad 0
\end{array}
$$

Exercise 5.4

Divide.

1. $x^2 + 7x + 12$	by	$x + 3$
2. $9x^2 + 6x + 1$	by	$3x + 1$
3. $3x^2 + 4x + 1$	by	$x + 1$
4. $a^2 - 8ab + 16b^2$	by	$a - 4b$
5. $6x^2 - 5x + 1$	by	$2x - 1$
6. $5x^2 + 7x - 12$	by	$x - 1$
7. $x^3 + 1$	by	$x + 1$
8. $5x^3 - 7x^2y - xy^2 + 3y^3$	by	$5x + 3y$
9. $x^3 - 3x^2y + 3xy^2 - y^3$	by	$x - y$

Review Exercise 5

1. Select the correct options in the following.

(i) For polynomials the exponents of the variables involved are:

(a) non negative integers (b) negative integers
(c) rational numbers (d) irrational numbers

(ii) A polynomial having two terms is called a:

(a) monomial (b) trinomial (c) binomial (d) constant

(iii) The degree of the polynomial $9x^3 - 7x + 5$ is:

(a) 0 (b) 1 (c) 2 (d) 3

(iv) The number of variables in the expression $yx^3 - 6xz + 5z$ are:

(a) 3 (b) 2 (c) 1 (d) 4

(v) The expression $6x^2 - 3x + 2$ is:

(a) linear (b) quadratic (c) cubic (d) constant

(vi) If A = $4x^2 + 6x + 9$ and B = $-2x^2 + 11x + 1$, then A + B is

(a) $2x^2 + 6x + 10$ **(b)** $6x^2 + 17x + 10$

(c) $2x^2 + 17x + 10$ **(d)** $6x^2 - 5x + 8$

(vii) $xy^2(2x + 8y^2)$ is equal to:

(a) $2x^2y^2 + 8xy^4$ **(b)** $2x^2y^2 + 8x^2y^3$

(c) $2xy^2 + 8x^2y^3$ **(d)** $2x^2y^2 + 8xy^3$

(viii) Product of $(x^2 + 2xy + y^2)$ and $(x + y)$ is equal to:

(a) $x^3 + y^3$ **(b)** $x^3 + 3x^2y + 3xy^2 + y^3$

(c) $x^3 - y^3$ **(d)** $x^3 + x^3y + xy^2 + y^2$

(ix) $[(x + a)^2 - (x - a)^2] \div 4ax$ = ...

(a) $4ax$ **(b)** 1 **(c)** $2x^2 + 2a^2$ **(d)** $2ax$

(x) $(a^2 - b^2)(a - b) \div (a + b)$ = ...

(a) $(a - b)$ **(b)** $a^2 + 2ab + b^2$

(c) $(a - b)^2$ **(d)** $(a + b)$

2. If A = $4x^4 + 6x^3 + 3x + 2$, B = $x^4 - 3x^3 + 5x - 2$ then find 2A + 4B.

3. Simplify $(5x - 3)(8x - 7) - (3x - 2)(5x - 1)$

4. Divide $a^3 + 3a^2b + 3ab^2 + b^3$ by a + b.

Key Point

- A symbol which has a fixed numerical value is called a constant.
- A quantity which has no fixed value but takes various numerical values is called a variable.
- Polynomials are algebraic expressions consisting of one or more terms in which exponents of the variables involved are non negative integers.
- A polynomial having one term is called a monomial. e.g., 3, x , 0 etc.
- A polynomial having two terms is called a binomial.
- A polynomial having three terms is called a trinomial.
- A polynomial having degree zero is called a constant polynomial.
- A polynomial having degree one is called a linear polynomial.
- A polynomial having degree two is called a quadratic polynomial.
- A polynomial having degree three is called a cubic polynomial.
- An algebraic expression consists of like and unlike terms, while adding and subtracting we collect and add the like terms.

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